Abstract
This paper presents the problem of identifying parameters for use in mathematical models of induction motors with the use of a genetic algorithm (GA). The effect of arithmetical crossover and the generation of new populations on identification results is analysed. The identified parameters of the model were determined as a result of the minimisation of the performance index defined as the mean-square error of stator current and angular velocity. The experiments were performed for the low power induction motor. The steady-state genetic algorithm with regard to convergence and accuracy of the identification process and calculation time is analysed.

Keywords: identification, mathematical model, induction motor, genetic algorithm, crossover

Streszczenie
W artykule przedstawiono problem identyfikacji parametrów modeli matematycznych silników indukcyjnych z zastosowaniem algorytmu genetycznego (AG). Analizowano wpływ krzyżowania arytmetycznego i generowania potomków na wyniki identyfikacji. Identyfikowane parametry modelu wyznaczono w rezultacie minimalizacji wskaźnika jakości zdefiniowanego jako błąd średniokwadratowy prądu stojącego i prędkości kątowej. Badania eksperymentalne przeprowadzono dla silnika indukcyjnego małej mocy. Algorytm genetyczny z częściową wymianą populacji analizowano ze względu na zbieżność i dokładność procesu identyfikacji i czas obliczeń numerycznych.

Słowa kluczowe: identyfikacja, model matematyczny, silnik indukcyjny, algorytm genetyczny, krzyżowanie
1. Introduction

Induction motors that are powered by voltage inverters are typically operated under variable load and power conditions, which affect the physical parameters of the motors and consequently, change their static and dynamic properties. These factors influence the transient and steady state of the motors and therefore the values determined in the process of identifying the parameters for the mathematical model [3, 14, 20].

Parametric identification methods of the induction motor mathematical model can be broadly divided into on-line and off-line strategies. Methods for off-line identification include classical static optimisation methods (such as Nelder-Mead’s method and Box’s method) and artificial intelligence (AI) methods (i.e. genetic/evolutionary/hybrid algorithms or artificial neural networks) [15, 18, 20]. The efficiency of AI methods make them increasingly popular both in technology and other fields; numerous examples are given in papers [2–11, 13]. AI methods are employed primarily when other methods do not offer a correct solution to a problem.

The parametric identification of induction motor is a very difficult problem and therefore the use of classical (numerical) optimisation methods is limited as the solutions of the mathematical model are unstable during the process. The local minimum of the performance index is determined, usually when the number of the identified parameters is large. It is then advisable to change the initial conditions or use another identification method, for example, a genetic algorithm that enables a high quality of the process for any starting conditions [17, 20].

The genetic algorithms (GAs) work on the population of individuals through the random choice of a sufficient number of representatives, the chance of determining the local minimum is smaller than in the case of classical methods [16, 20]. However, genetic algorithms demand more numerical calculations and the identification process is therefore time consuming [20].

Despite the abovementioned, attention should be paid to the structure of genetic algorithms with regard to such factors as representation of individuals, crossover, mutation, selection and the set of control parameters such as population size, probabilities of crossover, mutation and stop criterion [16]. Because its efficiency and effectiveness depends on it. It is reasonable to modify the genetic algorithm with regard to for example, selection modifications, crossover and mutation operators, elitist model introduction or the generation of new individuals [1, 4, 19, 21].

This paper attempts to substantially modify the genetic algorithm by controlling the crossover operator and generation of new individuals. The influence of the proposed genetic algorithm modifications on the identification results is analysed. A genetic algorithm based on floating point representation, tournament selection with steady-state, arithmetical crossover and uniform mutation is used. The experimental investigations are performed for an induction motor powered from voltage inverter.

In this work, a mathematical model of an induction motor in the rotating references frame, orientated according to the stator voltage vector was used. The parameters of the mathematical model of an induction motor were determined as a result of the minimisation of the performance index with the use of a genetic algorithm.
2. Formulation of the identification of the induction motor mathematical model

The model that takes into account the dynamics of the electromagnetic state and the dynamics of angular velocity is a complex system of nonlinear differential equations. Motor equations are presented in a rectangular coordinate system. It is common to use mathematical models of the motor formulated in the coordinates rotating in accordance with the stator voltage vector and in the stationary coordinate system [15, 16, 20].

The mathematical model of the induction motor in the reference frame $d$-$q$, orientated according to the voltage vector, has the following form [15]:

\[
\frac{d}{dt}\phi_d(t) = \phi_q(t)\omega_s(t) - R_s I_d(t) + v(t)
\]

\[
\frac{d}{dt}\phi_q(t) = -\phi_d(t)\omega_s(t) - R_s I_q(t)
\]

\[
\frac{d}{dt}I_d(t) = a_1\phi_d(t) + a_2\phi_q(t)\omega_s(t) - a_2 I_d(t) +
\]

\[
+ I_q(t)\omega_s(t) - I_q(t)\omega_s(t) + a_3 v(t)
\]

\[
\frac{d}{dt}I_q(t) = -a_3\phi_d(t)\omega_s(t) + a_1\phi_q(t) - I_d(t)\omega_s(t) +
\]

\[
+ I_d(t)\omega_s(t) - a_2 I_q(t)
\]

\[
\frac{d}{dt}\omega_s(t) = \frac{3p^2}{2J}\left(\phi_d(t)I_q(t) - \phi_q(t)I_d(t)\right) - \frac{P}{J}M_s(t)
\]

and:

\[
a_1 = \frac{R_r}{\sigma L_s L_r}, \quad a_2 = \frac{R_s}{\sigma L_s} + \frac{R_r}{\sigma L_r}, \quad a_3 = \frac{1}{\sigma L_s}
\]

\[
\sigma = \frac{L_s L_r - L_m^2}{L_s L_r}
\]

where:

- $I_d, I_q$ – components of stator current vector in the rotating $d$-$q$ reference frame,
- $\phi_d, \phi_q$ – components of stator flux vector,
- $L_s$ – inductance of stator,
- $L_r$ – inductance of rotor,
- $R_s$ – resistance of stator,
- $R_r$ – resistance of rotor,
\[ L_m \] – stator-rotor mutual inductance,
\[ \omega_e \] – electrical angular velocity and \( \omega_e = p \omega \),
\[ p \] – number of pole pairs,
\[ \omega_s \] – angular stator frequency,
\[ J \] – inertia moment of motor and load,
\[ M_o \] – load torque,
\[ v \] – module of stator voltage vector.

The schematic of the mathematical model of the induction motor with input and output signals is presented in Fig. 1. Input signals include the synchronous angular frequency \( \omega_s \) and the amplitude \( v \) of stator voltage vector; the output signals are angular velocity \( \omega \) and amplitude \( I \) of stator current vector [18].

Fig. 1. Schematic of the mathematical model of the induction motor with input and output signals

The selection of the performance index has a significant influence on the identification process results [18]. The identification of the mathematical model of the induction motor is more accurate if the performance index includes more than one component. If the performance index includes only one component, such as the stator current or angular velocity, the time responses of the selected quantity is consistent, but those of the quantities not included in the index may not overlap.

The parameter values of the mathematical model of the induction motor were determined as a result of the mean-square error minimisation of stator current \( I \) and angular velocity \( \omega \). The following identification performance index was assumed [16, 18]:

\[
Q = \frac{K}{N} \sum_{i=1}^{N} (I(i) - \hat{I}(i))^2 + \frac{1}{N} \sum_{i=1}^{N} (\omega(i) - \hat{\omega}(i))^2
\]  (3)

where:
\[ K \] – the weight coefficient experimentally determined in [16] in order to maintain the compromise between the mean-square error of stator current \( I \) and mean-square error of angular velocity \( \omega \),
\[ N \] – the number of measurements,
\[ ^\wedge \] – the solution of the motor mathematical model,
\[ I \] – the amplitude of the stator current vector.

This paper presents the minimisation of the performance index with the use of a genetic algorithm with steady-state. The author applied selected genetic/evolutionary algorithms to the identification problems. The papers [16–18] present important directions for further analysis of the effectiveness and efficiency of these algorithms.

This paper discusses the influence of arithmetical crossover and the generation of new individuals and populations on the results of the identification process, such as the identified
parameters and the time necessary for calculations. The steady-state genetic algorithm based on a floating point representation is used because this representation can be used even when the number of identified parameters is large. Every population is subject to genetic transformations such as the modified tournament selection, an arithmetical crossover and a uniform mutation. The arithmetical crossover is as follows [12]:

\[
\begin{align*}
    x'_1 &= ax_1 + (1-a)x_2 \\
    x'_2 &= ax_2 + (1-a)x_1
\end{align*}
\]  

where:
- \(x_1, x_2\) – parents,
- \(x'_1, x'_2\) – new individuals after crossover,
- \(a\) – crossover parameter \(a \in [0, 1]\), selected experimentally to obtain the best possible course of the genetic process.

The procedure for generating successive GA populations is implemented in such a way that in the current iteration of the algorithm, only one or two individuals of the previous population are exchanged. This is conditional on the modification of the crossover operator, where, depending on the approach applied, the number of descendants is changed. Depending on the value of the arithmetic crossover parameter \(a\), either one descendant that replaces the worst individual in the population or two descendants who replace their parents are created. Due to the small number of individuals exchanged, the algorithm requires a larger number of iterations because the concept of iteration is not identical to iteration, in which all or most of the population is exchanged.

Figure 2 presents the schematic of the mathematical model of the induction motor identification with the use of a genetic algorithm.

![Fig. 2. Schematic of induction motor parametric identification with the use of steady-state genetic algorithm](image-url)
3. Results of the parametric identification process

The identification of the mathematical model of the induction motor is based on the minimisation of the performance index with the use of a steady-state genetic algorithm. During the identification process, mathematical model parameters, such as: \( a_1 \), \( a_2 \), \( a_3 \), \( R_s \), \( R_r \) and \( J \) were determined. The following values of mathematical model parameters were accepted in computer simulations: \( a_1 = 521.4 \), \( a_2 = 280.1 \), \( a_3 = 54.2 \), \( R_s = 2.95 \, \Omega \), \( R_r = 2.47 \, \Omega \) and \( J = 0.04 \, \text{kgm}^2 \). The steady-state genetic algorithm was analysed with regard to convergence and accuracy of the identification process and the time of numerical calculations.

The genetic algorithms have a stochastic character, so every start of the identification procedure gives slightly different results. Thus, in experiments or simulations of identification, the average result is given. The influence of both arithmetical crossover and the generation of new individuals on the results of the simulation of the identification process with the use of a genetic algorithm is presented in Table 1. The values of identified parameters, the values of the performance index and the time of process are given as average results. In the tables, ‘\( a \)’ stands for the parameter of arithmetic crossover.

Table 1. The effect of arithmetical crossover and the generation of new individuals on the results of simulation of identification with the use of a steady-state genetic algorithm

<table>
<thead>
<tr>
<th>( a )</th>
<th>Identified parameters</th>
<th>Performance index</th>
<th>Time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( a_1 )</td>
<td>( a_2 )</td>
<td>( a_3 )</td>
</tr>
<tr>
<td>0</td>
<td>526.32</td>
<td>283.32</td>
<td>57.14</td>
</tr>
<tr>
<td>0.3</td>
<td>525.54</td>
<td>284.98</td>
<td>57.33</td>
</tr>
<tr>
<td>0.5</td>
<td>522.81</td>
<td>281.62</td>
<td>55.61</td>
</tr>
<tr>
<td>Random</td>
<td>526.22</td>
<td>282.65</td>
<td>57.99</td>
</tr>
<tr>
<td>1</td>
<td>527.84</td>
<td>283.65</td>
<td>58.02</td>
</tr>
</tbody>
</table>

The results of experimental identification of the mathematical model of the induction motor with the use of a steady-state genetic algorithm are shown in Table 2. The effect of the arithmetical crossover and the generation of new individuals on the identification results and calculation time is also analysed. The convergence of time responses of motor angular velocity and current with the mathematical model is assessed by multidimensional correlation factors \( R_{\omega} \) and \( R_I \).

The results of simulations and experiments show that the value of crossover parameter \( a \) considerably influences both the values of the identified parameters and the time duration of the process. The parameter of arithmetical crossover \( a = 0.5 \) ensured the most accurate results (due to the values of identified parameters and the time of the process), whereas the least accurate results were obtained for operator of crossover values \( a = 0 \) and \( a = 1 \). The application of \( a = 0.3 \) and \( a \)-random may, in some cases, yield comparable results. The value of crossover parameter
a also affects the number of created descendants and the creation of a new population. For $a = 0.5$, only one new individual replacing the worst individual in the population is created. For $a = 0$ and $a = 1$, two new individuals that are generally the same as their parents are created; thus, mutation plays an important role when a new population is entered.

Table 2. The influence of arithmetical crossover and the generation of a new population on the results of experimental identification with the use of a steady-state genetic algorithm

<table>
<thead>
<tr>
<th>$a$</th>
<th>Identified parameters</th>
<th>Performance index</th>
<th>$R_\alpha$</th>
<th>$R_\beta$</th>
<th>Time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$a_1$</td>
<td>$a_2$</td>
<td>$a_3$</td>
<td>$R_\alpha$</td>
<td>$R_\beta$</td>
</tr>
<tr>
<td>0</td>
<td>537.18</td>
<td>289.53</td>
<td>59.32</td>
<td>3.56</td>
<td>2.89</td>
</tr>
<tr>
<td>0.3</td>
<td>527.35</td>
<td>285.32</td>
<td>58.40</td>
<td>3.39</td>
<td>2.36</td>
</tr>
<tr>
<td>0.5</td>
<td>525.54</td>
<td>282.12</td>
<td>56.28</td>
<td>3.34</td>
<td>2.46</td>
</tr>
<tr>
<td>Random</td>
<td>519.83</td>
<td>286.08</td>
<td>59.22</td>
<td>2.88</td>
<td>2.66</td>
</tr>
<tr>
<td>1</td>
<td>535.21</td>
<td>290.35</td>
<td>59.00</td>
<td>3.46</td>
<td>2.87</td>
</tr>
</tbody>
</table>

In Fig. 3, time responses of the induction motor obtained in the experimental identification process with the use of the genetic algorithm are compared to those of its mathematical model. It can be seen that a very strong convergence between motor time responses and the time responses of its mathematical model was obtained.

![Fig. 3. Comparison of time responses of induction motor (solid line) and its mathematical model (dashed line) in the identification process with the use of the genetic algorithm (the identified parameters: $a_1$, $a_2$, $a_3$, $R_\alpha$, $R_\beta$, $J$)](image)

### 4. Conclusions

The paper has presented the parametric identification of a mathematical model of an induction motor with the use of a steady-state genetic algorithm. The effect of arithmetical crossover and the generation of new populations on the identification results has been analysed.
On the basis of simulation and experimental investigations, it has been confirmed that the adopted parameter of arithmetical crossover significantly affects the genetic process. It is claimed that the proposed crossover (parameter of arithmetical crossover $a = 0.5$) in which only one new individual replacing the worst individual is created allows us to get the most accurate identification results considering the value of the identified parameters and the time of the process. Assuming that the value of $a = 0$ or $a = 1$, where two descendants, almost identical to their parents whom they replace in the population, the least accurate results are obtained. In these cases, the operation of mutation has a decisive influence on the solution. The random value of $a$ or at the level of 0.3 makes it possible to statistically achieve a good convergence between motor time responses and the time responses of its mathematical model; however, the identification process takes longer than that for which the parameter $a = 0.5$.

References


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