Analysis of the crack width of beams reinforced with FRP bars

Analiza zarysowania belek zbrojonych prętami FRP

Abstract
The paper investigates and compares a selected issue of Serviceability Limit State (SLS) of simply supported reinforced concrete (RC) beams subjected to various values of flexural stresses. Characteristic crack widths of beams reinforced with various types of bars were calculated. Beams reinforced with Glass Fiber Reinforced Polymer (GFRP), Carbon Fiber Reinforced Polymer (CFRP) and Aramid Fiber Reinforced Polymer (AFRP) were examined. The computational analysis of beams reinforced with FRP bars was based on Italian guideline for the design (CNR-DT 203/2006) and in accordance with the EC2 (EN 1992-1-1:2004). Based on the conducted analysis, the effect of changing the service live load on the increase in crack width was presented.

Keywords: FRP reinforcement, RC beam, crack width, design recommendations, CNR-DT 203/2006

Streszczenie

Słowa kluczowe: zbrojenie FRP, belka zbrojona, szerokość rozwarcia rysy, zalecenia projektowe, CNR-DT 203/2006
1. Introduction

Recently, an increased interest in the use of the fiber reinforced polymer (FRP) materials in building and engineering structures can be observed [1–4]. Among other things, the use of FRP rebar as the main reinforcement of concrete structures is increasingly common. This interest arises due to very good physical and mechanical characteristics of this material. Especially, low self-weight, good fatigue properties (parallel to the fibers orientation) due to the production method (pultrusion) [5, 6], very high resistance to the impact of aggressive environmental factors and high corrosion resistance generate intensified research of possibility to use FRP bars [1–3, 7–11]. Such material characteristics of a FRP rebar as strength and stiffness properties depend on the type of used fiber. There are FRP bars reinforced with the following types of fibers [2]: glass (GFRP), carbon (CFRP), aramid (AFRP) and basalt (BFRP).

The main purpose of this paper is to determine the possibility of using FRP reinforcement. Accordingly, a computational analysis of simply supported reinforced concrete beams affected by various values of flexural stresses was performed. The analysis consists in the comparison of results obtained for RC beams reinforced with FRP rebars. During the research, crack widths were taken into consideration. The results of the analysis allow for the specification of the most effective reinforcement under assumed conditions.

2. Computational analysis

It is worth noting that there is no available standard for strength calculations of sections reinforced with FRP rebars [11–13]. As a result of long-term experience of many researchers and designers, guidelines for the design of structural elements reinforced with FRP bars were elaborated. Among the available recommendations for design, four most popular guidelines could be distinguished: the Italian [14], the American [15], the Canadian [16] and the Japanese ones [17]. These design recommendations have been developed on the basis of analytical solutions and empirical equations, which are supported by experimental tests carried out on FRP bar samples and structural RC elements reinforced with FRP rebars [1–4, 12, 13]. The computational analysis of beams was made on the basis of applicable standards and design recommendations.

2.1. Calculation model

Simply supported beams with a rectangular cross-section were assumed. The outline of the static scheme and cross-section of the investigated beams are presented in Fig. 1. The dimensions of the cross-section are the following: $b = 180$ mm, $h = 350$ mm, and the effective length of the span is $L_{eff} = 4.0$ m. The beams are reinforced with $3\Phi 12$ of the following bars: CFRP, ARFP and GFRP. The used concrete is specified by C20/25 compressive strength class and the concrete cover thickness is assumed as $c = 35$ mm. The
material characteristics of concrete were as follows: the characteristic compressive strength $f_{ck} = 20$ MPa, the mean value of axial tensile strength of concrete $f_{ctm} = 2.2$ MPa, the ultimate compressive strain in the concrete $\varepsilon_{cu} = 0.0035$ and the modulus of elasticity of normal weight concrete $E_{cm} = 30 \cdot 10^3$ MPa. The strength properties of the rebar are reported in accordance with the bar manufacturer’s material data [18, 19]. The beam is designed to carry a service dead load equal to $w_{SDL} = 3.0$ kN/m and a various value of a service live load equal to $w_L = \{0.5; 2.0; 3.5; 5.0; 6.5; 8.0; 9.5; 11.0\}$ kN/m. The limitation of the crack widths is assumed as $w_{k,lim} = 0.4$ mm, in accordance with the EC2 standard [20]. The beam is not exposed to the moisture.

2.2. Calculation method

The calculation method of beams reinforced with FRP was chosen in accordance with the Italian design recommendations [14], because of similarity thereof to the EC2 [20]. The material characteristics and environmental conversion factors of FRPs are specified by the manufacturer [18, 19] and shown in Table 1.

<table>
<thead>
<tr>
<th>Type of rebar</th>
<th>The characteristic tensile strength of FRP $f_{tk}$ [MPa]</th>
<th>The modulus of elasticity of FRP $E_f$ [MPa]</th>
<th>The design strain of FRP $\varepsilon_{fd}$ [-]</th>
<th>The environmental conversion factor $\eta_a$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFRP</td>
<td>2 300</td>
<td>130-10^3</td>
<td>0.018</td>
<td>1.0</td>
</tr>
<tr>
<td>AFRP</td>
<td>1 400</td>
<td>60-10^3</td>
<td>0.023</td>
<td>0.9</td>
</tr>
<tr>
<td>GFRP</td>
<td>1 300</td>
<td>55-10^3</td>
<td>0.022</td>
<td>0.8</td>
</tr>
</tbody>
</table>

The characteristic crack widths $w_k$ [mm] of the FRP reinforced element were calculated according to the Italian guidelines [14] using the following equation:

$$w_k = \beta w_{k,lim}$$  \hspace{1cm} (1)

and compared with the ultimate value of the crack widths $w_{k,lim}$. The non-dimensional coefficient $\beta$ is relating the average crack width to the characteristic value and is assumed according to the design recommendation [14]. The final average distance between cracks $s_{rm}$ [mm] was calculated using Eq. 2. It depends on bond properties of the FRP bars (non-
dimensional coefficient \( k_1 \) and the strain diagram (non-dimensional coefficient \( k_2 \)). The suitability of the relationships provided by the EC2 standard \([20]\) for computations of the distance between cracks was assumed.

\[
s_{rm} = 50 + 0.25 \cdot k_1 \cdot k_2 \frac{d_b}{\rho_r}
\]  

where:

- \( d_b \) – the equivalent diameter of the FRP reinforcement, in mm;
- \( \rho_r \) – the effective reinforcement ratio, equal to \( \frac{A_f}{A_{c,eff}} \), where \( A_{c,eff} \) is the effective concrete area in tension defined according to the EC2 standard \([20]\) (the area around the tensile FRP reinforcement, which has a depth equal to the distance between FRP’s centroid and tension fiber of concrete multiplied by 2.5).

The average strain accounting for tension stiffening, shrinkage, etc. is defined by \( \varepsilon_{fm} \) (Eq. 3). It depends on non-dimensional coefficients \( \beta_1 \) (accounting for bond properties of FRPs) and \( \beta_2 \) (accounting for the duration of loading).

\[
\varepsilon_{fm} = \varepsilon_f \cdot \left( 1 - \beta_1 \cdot \beta_2 \cdot \frac{\sigma_{fr}^{m}}{\sigma_f^{m}} \right)
\]  

where:

- \( \varepsilon_f \) – the strain of FRP reinforcement;
- \( \sigma_f \) – the FRP reinforcement stress in tension of the cracked cross-section, in MPa;
- \( \sigma_{fr} \) – the FRP reinforcement stress in tension of the cracked cross-section (when the first crack is observed), in MPa;
- \( m \) – the coefficient, which equals 2.

2.3. The results of the analysis

The results of the characteristic crack widths \( w_k \) of the beams obtained during the theoretical analysis were compared in Table 2.

<table>
<thead>
<tr>
<th>The live load level ( w_L ) [kN/m]</th>
<th>0.5</th>
<th>2.0</th>
<th>3.5</th>
<th>5.0</th>
<th>6.5</th>
<th>8.0</th>
<th>9.5</th>
<th>11.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of rebar</td>
<td>CFRP</td>
<td>0.006</td>
<td>0.182</td>
<td>0.259</td>
<td>0.299</td>
<td>0.322</td>
<td>0.337</td>
<td>0.347</td>
</tr>
<tr>
<td></td>
<td>AFRP</td>
<td>0.025</td>
<td>0.753</td>
<td>1.069</td>
<td>1.235</td>
<td>1.332</td>
<td>1.394</td>
<td>1.436</td>
</tr>
<tr>
<td></td>
<td>GFRP</td>
<td>0.034</td>
<td>1.020</td>
<td>1.448</td>
<td>1.673</td>
<td>1.804</td>
<td>1.888</td>
<td>1.945</td>
</tr>
</tbody>
</table>

Assuming that the limitation of the crack widths equals 0.4 mm, only beams reinforced with CFRP bars satisfied the condition. The beams reinforced with AFRP and GFRP bars
exceed the limit value already at live load level equal to $w_L = 2.0 \text{kN/m}$. The reason of this phenomena is low values of modulus of elasticity of polymer rebar (AFRP and GFRP). The modulus of elasticity of CFRP is about two times higher than the ones of the other FRPs.

Interestingly, the increase in crack width of the beams reinforced with FRP seems to change as the logarithmical function. It means that the crack width growth at the initial load increase is more dynamic.

3. Summary

In this paper, an attempt to determine the most effective FRP reinforcement of a RC beam under assumed conditions was made. The characteristic crack widths which depend on the various level of service live load were calculated and compared. The calculations were done in accordance with the applicable standards and design recommendations [14-17, 20].

As it was suspected, during the computational analysis it was observed that the value of modulus of elasticity of the FRPs has the greatest impact on the growth of crack widths. Only beams reinforced with CFRP bars satisfy the condition of crack width limitations. It was noticed that the crack widths growth increased logarithmically as far as the intensity level of the acting live load increased. This is interesting due to the fact that in the case of steel reinforcement, as indicated by calculations made in accordance with EC2 [20], the crack width increased linearly. The obtained results will allow us to understand the flexural behavior of FRP RC members within SLS. In order to fully determine the effectiveness of FRP reinforcement under the SLS, further analysis is recommended.

References