TECHNICAL TRANSACTIONS 12/2018
COMPUTER SCIENCES

DOI: 10.4467/2353737XCT.18.182.9670
SUBMISSION OF THE FINAL VERSION: 27/11/2018

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Reactive T-topology four-terminal-network compensator for multiharmonic current

Reaktancyjny kompensator czwórnikowy kształtu T dla wieloharmonicznego źródła rzeczywistego

Abstract

This paper presents a new method for passive current compensation of a real multiharmonic power source with the use of a four-terminal network. In contrast to a two-terminal compensator, a four-terminal-network compensator can fully separate the supply circuit from the load. This ensures optimal operating conditions for the source while keeping the voltage and load current unchanged. The source's optimal working conditions mean that the source current reaches its minimal RMS value (becoming the so-called "active current") while it transmits the desired active power to the load.

Keywords: active power, optimal current, multiharmonic current

Streszczenie

W artykule przedstawiono metodę kompensacji biernego prądu źródła wieloharmonicznego rzeczywistego przy pomocy układu czwórnikowego. Kompensator czwórnikowy, w przeciwieństwie do dwójnikowego, potrafi odseparować obwód zasilania od obwodu odbiornika i zapewnić źródłu optymalne warunki pracy przy jednoczesnym zachowaniu niezmienionych wartości napięcia i prądu odbiornika. Przez optymalne warunki pracy źródła należy tu rozumieć całkowite zminimalizowanie wartości skutecznej prądu źródła zasilania do tzw. prądu aktywnego niosącego zadaną moc czynną ze źródła do odbiornika.

Słowa kluczowe: moc czynna, prąd optymalny, źródło wieloharmoniczne

1. Introduction

In the case of an AC power-supplying source with an internal impedance, the commonly used reactive current compensation based on one capacitor in parallel does not minimize transmission losses or the RMS value of the source current. When the known equivalent circuit parameters of the energy source are taken into account, minimization of the source current RMS value and transmission losses cannot be achieved with the use of a passive two-terminal-network compensator. This paper presents a synthesis of a four-terminal-network compensator that provides optimal operating conditions for a source and nominal operating conditions for a load using only reactive elements.

2. Optimal operating conditions – monoharmonic case

The optimal operating condition of a source is usually formulated as achieving the minimum RMS value of the source's current (minimal transmission losses), under assumption that a given active power is supplied to the load, which can be formulated as the following constrained minimization task [3, 4]:

$$\mathrm{II}^{*} \rightarrow \min$$

$$P = \mathrm{real}\, I^{*}E - R_S I^{*} I \tag{1}$$

where:

$$R_S = 0.5\left(Z_S + Z_S^{*}\right)$$

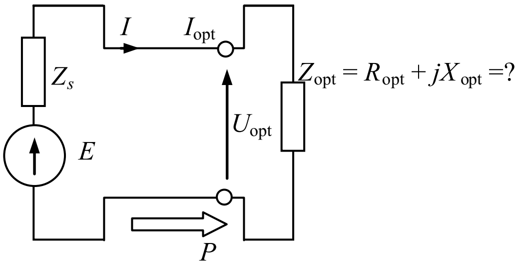

Fig. 1. The minimization task

The solution to this problem is a current which is in phase with the source's EMF [1–3]

$$I = G_e E \tag{2}$$

where G_e stands for an equivalent conductance seen by the EMF.

Additionally, this current should meet the power equation

$$R_S I^{*} I - \mathrm{real}(I^{*}E) + P = 0$$

or

$$R_s G_e^2 - G_e + \frac{P}{|E|^2} = 0$$

which allows us to calculate the optimal conductivity

$$G_e = \frac{1 - \sqrt{1 - \frac{P}{P_{max}}}}{2R_s} \tag{3}$$

and then the optimal current

$$I_{opt} = G_e E = \frac{1 - \sqrt{1 - \frac{P}{P_{max}}}}{2R_s} E \tag{4}$$

where

$$P_{max} = \frac{|E|^2}{4R_s} \tag{5}$$

3. A four-terminal reactive compensator

A passive four-terminal compensator can be created using reactive elements. It must meet the optimum input condition and nominal load condition.

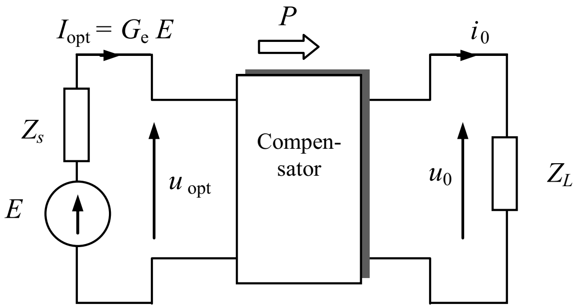

Fig. 2. Four-terminal compensator

Therefore, to determine its parameters it is enough to know the optimum input conditions (I_{opt} and U_{opt}) and load impedance Z_L.

From the minimum current condition under the active-power constraint (formula 1) [3, 4], we obtain the value of the equivalent conductivity G_e [1, 2, 5]; thus, from the EMF's point of view, the entire system reduces to a resistance equal to $\frac{1}{G_e}$, which in turn amounts to impedance Z_{opt} in series with inner impedance Z_s (see Fig 3).

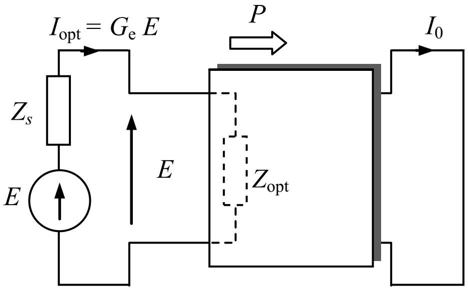

Fig. 3. Equivalent system seen by the EMF after optimization

4. T-topology compensator – monoharmonic case

One possible implementation of a four-terminal reactive compensator is with the use of the T-topology four-terminal network shown in figure 4, which acts as a matching circuit that connects the source and the load.

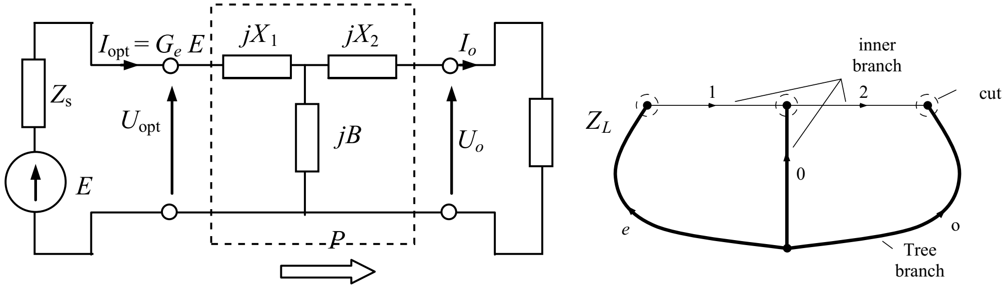

Fig. 4. Left – reactive compensator as a T-topology four-terminal network; right – graph of the circuit

Such a compensator must have at least 3 branches because then we have one independent current or voltage within the circuit's inner branches and this gives us some choice in the selection of reactances [6].

When describing the system with the use of chain equations, we get the following formulas:

$$\begin{bmatrix} U_{\text{opt}} \\ I_{\text{opt}} \end{bmatrix} = \begin{bmatrix} 1 & jX_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ jB & 1 \end{bmatrix} \begin{bmatrix} 1 & jX_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ I_o \end{bmatrix} = \\ = \begin{bmatrix} 1 - X_1 B & j(X_1 + X_2 - X_1 X_2 B) \\ jB & 1 - X_2 B \end{bmatrix} \begin{bmatrix} 1 & Z_L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ I_o \end{bmatrix} \tag{6}$$

or

$$\begin{bmatrix} U_{\text{opt}} \\ I_{\text{opt}} \end{bmatrix} = \begin{bmatrix} a & jb \\ jc & d \end{bmatrix} \begin{bmatrix} 1 & R_L + jX_L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ I_o \end{bmatrix} A \begin{bmatrix} 0 \\ I_o \end{bmatrix} \tag{7}$$

where $Z_L = R_L + jX_L$

Thus, the optimal input impedance is:

$$Z_{\text{opt}} = \frac{A_{12}}{A_{22}} = R_{\text{opt}} + j\,X_{\text{opt}} \tag{8}$$

Meeting the optimal input condition requires that the impedance seen by the SEM is real and equal to $1/G_e$. This condition imposes two equations:

$$R_s + R_{\text{opt}} = R_s + \frac{R_L}{c^2(R_L^2 + X_L^2) + d^2 - 2X_L cd} = \frac{1}{G_e}$$

$$X_s + X_{\text{opt}} = X_s - \frac{ac(R_L^2 + X_L^2) + X_L(bc - ad) - bd}{c^2(R_L^2 + X_L^2) + d^2 - 2X_L cd} = 0 \tag{9}$$

Since the compensator has three reactances, the solution of these two equations must depend on the third compensator's parameter, e.g. the B-susceptance, which in a certain range (around zero) gives a real solution for X_1 and X_2, but only if

$$-\frac{1}{\sqrt{R_L(1/Ge - Rs)}} < B < \frac{1}{\sqrt{R_L(1/Ge - Rs)}}$$

since always $R_s G_e < 1$, thus

$$X_1(B) = -X_S + \frac{1}{B} \pm R_{\text{opt}}\sqrt{\frac{1}{R_{\text{opt}} B^2 R_L} - 1}$$

$$X_2(B) = -X_L + \frac{1}{B} \pm R_L\sqrt{\frac{1}{R_{\text{opt}} B^2 R_L} - 1} \tag{10}$$

5. A distorted, multi-harmonic source's current

In the case of a multi-harmonic source current, the optimization task (1) takes the form:

$$\sum_h I_h I_h^* \to \min$$

$$P = \sum_h \text{real } I_h^* E_h - \sum_h R_{S,h} I_h^* I_h \tag{11}$$

where $h = 1 \ldots N$. Its solution can be obtained in a similar way as in [3, 4, 5] by minimizing the following functional

$$\sum_h I_h I_h^* - \frac{1}{\lambda}\left(\sum_h \text{real } I_h^* E_h - \sum_h R_{S,h} I_h^* I_h\right) \to \min$$

for $\lambda > 0$, we get

$$I_h = \frac{E_h}{2(R_{S,h} + \lambda)} = \frac{E_h}{R_{S,h} + R_{\text{opt},h}} = G_{e,h} E_h$$

and the transmitted P power condition can be written as

$$\begin{aligned} P(\lambda) &= \sum_h R_{L,h} \frac{|E_h|^2}{\left|Z_{S,h} + Z_{L,h}\right|^2} = \\ &= \sum_h \frac{|E_h|^2}{2(R_{S,h} + \lambda)} - \sum_h R_{S,h} \frac{|E_h|^2}{4(R_{S,h} + \lambda)^2} \end{aligned} \tag{12}$$

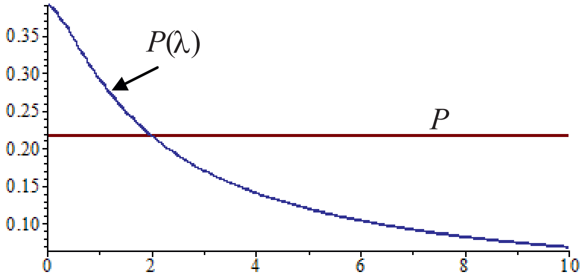

Fig. 5. An example of the variability of $P(\lambda)$ and imposed P value

As can be seen from the previous equations (9), the optimal SEM's load for each harmonic fulfills the resonance condition

$$X_{S,h} + X_{\text{opt},h} = 0$$

Equation (9) now takes the form

$$R_{\text{opt},h} = \left[\frac{R_L}{c^2(R_L^2 + X_L^2) + d^2 - 2X_L cd} \right]_h \tag{13}$$

$$X_{s,h} = \left[\frac{ac(R_L^2 + X_L^2) + X_L(bc - ad) - bd}{c^2(R_L^2 + X_L^2) + d^2 - 2X_L cd} \right]_h$$

For each harmonic we get

$$X_1(B,h) = -X_{S,h} + \frac{1}{B_h} \pm R_{opt,h} \sqrt{\frac{1}{B_h^2 R_{L,h} R_{opt,h}} - 1} \tag{14}$$

$$X_2(B,h) = -X_{L,h} + \frac{1}{B_h} \pm R_{L,h} \sqrt{\frac{1}{B_h^2 R_{L,h} R_{opt,h}} - 1}$$

Where for every h

$$R_{\text{opr},h} = \frac{1}{G_{e,h}} - R_{E,h} > 0$$

The solution is real only if for each harmonic:

$$\frac{1}{\sqrt{R_{L.h}(1/G_{e,h} - R_{E,h})}} < B_h < \frac{1}{\sqrt{R_{L.h}(1/G_{e,h} - R_{E,h})}}$$

because always $R_{E,h}G_{e,h} < 1$

After determining the λ value from the non-linear equations (12), the $R_{\text{opt},h}$ is then

$$R_{\text{opt},h} = R_{S,h} + 2\lambda \tag{15}$$

6. T-topology compensator – a multi-harmonic case

The harmonic-dependent susceptance B and reactance X_1, X_2 of the four-terminal T-topology compensator shown in Figure 4 can be realized as LC filters:

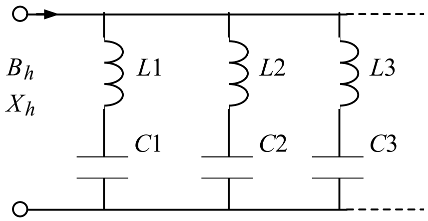

Fig. 6. Implementation of a compensator branch as an LC filter

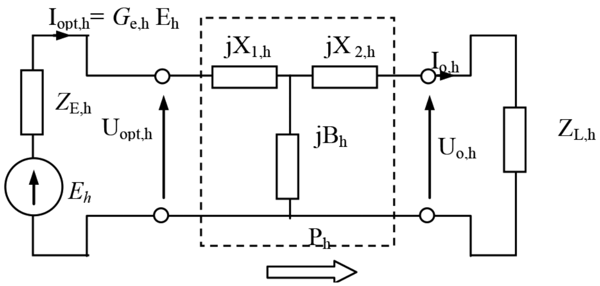

Fig. 7. T-topology four-terminal compensator in the case of multiharmonic current

The susceptance of such a filter has the form

$$B_h = \sum_{n \in 1..g} \omega_1 C_n \frac{h}{1-(h/h_n)^2}$$

where:

h – harmonic number (natural number),

g – number of the filter's parallel branches,

h_n – harmonic resonance of n's branch,

$\omega_1 = 2\pi f_1$,

f_1 – fundamental frequency.

7. Calculation example

Let us consider the optimization task of the two-harmonic source current of the system depicted in Fig. 8 with an RL load

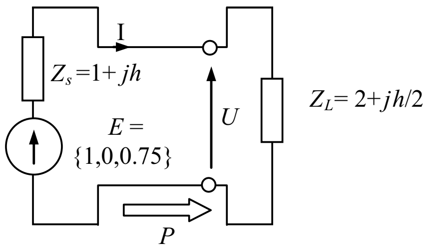

Fig. 8. Two-harmonic power source and its load before optimization

Before optimization:

$$|I| = 0.328$$
$$|U| = 0.70$$
$$P = 0.216$$

Solving (12) (Fig. 5) we get $\lambda = 2$, thus $R_{opt} = 5\ \Omega$ (which is constant due to the constant resistance R_S of the source).

Two solutions of $X_{1,h}$ and $X_{2,h}$ that depend on B_h (14) are thus possible; we choose one of them assuming inductive susceptance $B_h \sim \dfrac{1}{h}$

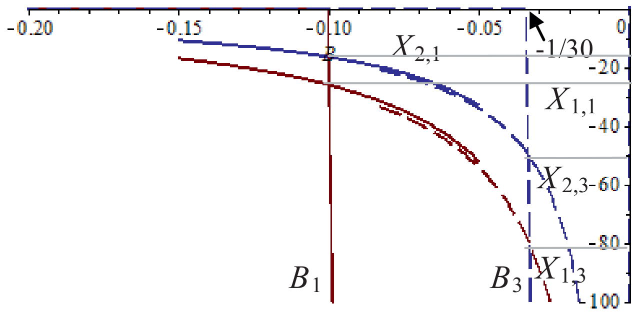

Fig. 9. Example relations of X_1, X_2 with respect to B (L = 31.85 [mH])

We select $B_1 = -0.1$ and $B_3 = -1/30$, thus capacitive reactances are

$$X_{1,1} = -26.0 \quad X_{1,3} = -80.3$$
$$X_{2,1} = -16.5 \quad X_{2,3} = -50.3$$

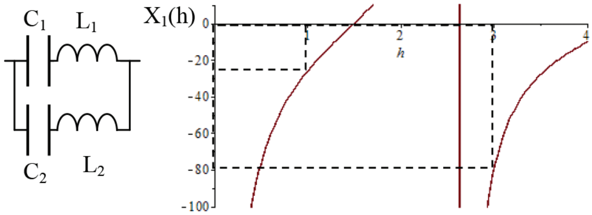

Fig. 10. implementation of $X_{1,h}$ and its frequency response

Thus, 1st branch parameters are i.e.:

$$C_1 = 57.4 \quad C_2 = 17.9\ [\mu F]$$
$$L_1 = 78.5 \quad L_2 = 27.8\ [mH]$$

The T-topology four-terminal compensator is depicted in Fig. 11

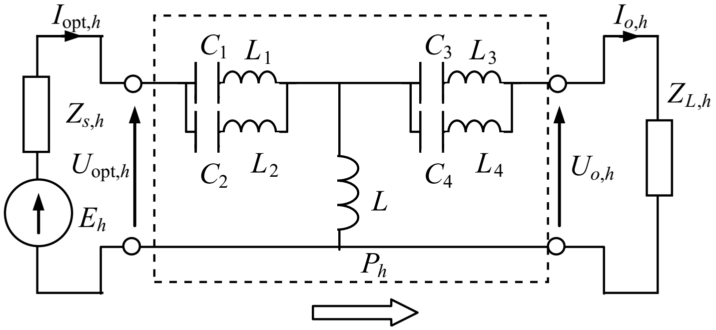

Fig. 11. T-topology four-terminal compensator

After connecting the compensator

$$|I_{opt}| = 0.207$$
$$|U_{opt}| = 1.12$$
$$|U_o| = 0.7$$
$$P = 0.216$$

Thus, a lossless and reactive four-terminal compensator that completely compensates the source's reactive current is feasible and is shown in Fig. 11.

8. Summary

The lossless, four-terminal compensator makes the power-source's voltage and current independent of the load's voltage and current, which makes it possible to ensure optimal operating conditions for the source without any need to modify the load's operating conditions (voltage, current), which is unavoidable in the parallel or serial compensation method. In addition, because of its zero active power branches, such a four-terminal matching compensator can be implemented in the future with the use of so-called 'active filters'.

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If you want to quote this article, its proper bibliographic entry is as follow: Jaraczewski M., *Reactive t-topology four-terminal-network compensator for multiharmonic current*, Vol. 12/2018, pp. 117–126.