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ESTIMATING POTENTIAL LOSSES OF THE CLIENT IN PUBLIC PROCUREMENT IN CASE OF COLLUSION UTILIZING A MLP NEURAL NETWORKS

WYKORZYSTANIE SZTUCZNYCH SIECI NEURONOWYCH DO OSZACOWANIA POTENCJALNYCH STRAT ZAMAWIAJĄCEGO W WYNIKU NIEKONKURENCYJNYCH ZACHOWAŃ OFERENTÓW

Abstract

There are two types of collusion which can harm Clients. This paper proposes two methods of detecting collusion's and estimation a clients' potential losses. The first is based on officially recommended factors for collusion detecting. The second one utilizes MLP artificial neural networks. Results are compared and discussed.

Keywords: collusion MLP artificial neural networks public procurements

Streszczenie

W artykule scharakteryzowano rodzaje zmów przetargowych prowadzących w efekcie do strat zamawiających w zamówieniach publicznych. Zaproponowano 2 metody wykrywania zmów i oszacowania strat zamawiających, strat powstałych w wyniku zmywu. Pierwszą oparto o oficjalnie opublikowane wskaźniki do detekcji zmów przetargowych. Druga bazuje na wykorzystaniu sztucznych sieci neuronowych MLP. Otrzymane wyniki porównano i przedyskutowano.

Słowa kluczowe: zmywy przetargowe sztuczne sieci neuronowe MLP zamówienia publiczne

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1. Introduction

The main goal when selecting a supplier at the tendering stage is to ensure that the offer is favorable in financial terms. This is possible when many independent entrepreneurs take part in the procedure who are not related to each other and who do not know each others bids [1]. Only under these terms bidders have motivation to lower prices hoping to defeat other rivals and win the contract.

Bid rigging (or collusive tendering) occurs when businesses, that would otherwise be expected to compete, secretly conspire to raise prices or lower the quality of goods or services for purchasers who wish to acquire products or services through a bidding process. In order to get the best offer, public and private organizations often use procurement procedures carried out in conditions of competition. Low prices and/or better products provide savings or lead to freeing up resources which can be used for other purposes [2]. Competition should lower prices and improve quality, but only on the condition that companies compete fairly (i.e. construct their offers in a proper and independent way). The practice of bid rigging can be especially harmful in public sector tenders. This type of collusion deprives authorities' and taxpayers' of money and contributes to the reduction of public trust in relation to the government. What is more, it reduces the benefits of a competitive market.

In all the countries belonging to the OECD bid rigging is against the law and is subject to investigation and sanctions in accordance with the rules of competition law. In many OECD countries, this practice is a crime.

Numerous studies into bid collusion at auctions have been conducted in the past. Empirical literature is generally focused on detecting of collusion using past examples. Porter and Zona [5], Zona [6] propose tests for collusion detection in the context of highway construction data and school milk markets. Baldwin *et al* examine competitive and non-competitive behaviors using forest timber sales data. Howard, Kasermann [8] and Nelson [9] calculate damages forecasts where bid-rigging has occurred. McMillan [10] conducts a similar study for Japanese public contracts. Pesendorfer [11] uses data for school milk contracts at first-price sealed-bid auctions during the 1980s as a case study for collusive behavior in auctions. Results of the research conducted in the studies mentioned above were used by the European Commission to create collusion-related documents directive proposal [3] and the document on quantifying harm [4].

2. Actual state of law in Poland

The basic legal act that regulates issues connected with competition protection in Poland is the Act of 16th February 2007 on Competition and Consumer Protection also called the Anti-trust Act. According to this act companies are not allowed to perform any actions, which could limit the competition. It includes companies that compete directly with each other at the same level of trade (horizontal agreement) and non-competing companies operating in different trade levels (vertical agreement) [1].

3. Types of collusive behavior

Although individuals and firms may agree to implement bid-rigging schemes in a variety of ways, they typically implement one or more of several common strategies. These techniques are not mutually exclusive. For example, cover bidding may be used in conjunction with a bid-rotation scheme [2]. These strategies in turn may result in patterns that procurement officials can detect and which can then help uncover bid-rigging schemes.

3.1. Cover bidding

Cover (also called complementary, courtesy, token or symbolic) bidding is the most frequent way in which bid-rigging schemes are implemented. It occurs when individuals or firms agree to submit bids that involve at least one of the following: (1) a competitor agrees to submit a bid that is higher than the bid of the designated winner, (2) a competitor submits a bid that is known to be too high to be accepted, or (3) a competitor submits a bid that contains special terms that are known to be unacceptable to the purchaser. Cover bidding is designed to give the appearance of genuine competition.

3.2. Bid suppression

Bid-suppression schemes involve agreements among competitors in which one or more companies agree to refrain from bidding or to withdraw a previously submitted bid so that the designated winner's bid will be accepted. In essence, bid suppression means that a company does not submit a bid for final consideration [2].

3.3. Bid rotation

In bid-rotation schemes, conspiring firms continue to bid, but they agree to take turns being the winning (i.e., lowest qualifying) bidder. The way in which bid-rotation agreements are implemented can vary [1, 2]. For example, conspirators might choose to allocate approximately equal monetary values from a certain group of contracts to each firm or to allocate volumes that correspond to the size of each company.

3.4. Market allocation

Competitors divide the market and agree not to compete for certain customers or in certain geographic areas. Competing firms may, for example, allocate specific customers or types of customers to different firms, so that competitors will not bid (or will submit only a cover bid) on contracts offered by a certain class of potential customers which are allocated to a specific firm [1, 2]. In return, that competitor will not competitively bid to a designated group of customers allocated to other firms in the agreement.

4. Quantifying harm in competition cases

Everyone who has suffered harm because of an infringement of Article 101 or 102 of the Treaty on the Functioning of the European Union (TFEU) has a right to be compensated for that harm. The Court of Justice of the EU held that this right is guaranteed by primary EU law 1. Compensation means placing the injured party in the position it would have been in had there been no infringement. Therefore, compensation includes reparation not only for actual loss suffered (*damnum emergens*), but also for loss of profit (*lucrum cessans*) and the payment of interest [4]. Actual loss means a reduction in a person's assets; loss of profit means that an increase in those assets, which would have occurred without the infringement, did not happen. Proving and quantifying antitrust harm is generally very fact-intensive and costly, as it may require the application of complex economic models [3]. Methods that can be used to quantify potential harm are listed below:

4.1. Comparison over time on the same market

One frequently used method consists in comparing the actual situation during the period when the infringement produced effects with the situation on the same market before the infringement produced effects or after they ceased [3].

4.2. Comparison with data from other geographic markets

This may be data observed across the entire geographic comparator market or data observed in relation to certain market participants only [4]. The more similar a geographic market is (except for the infringement effects) to the market affected by the infringement, the more it is likely to be suitable as a comparator market. This means that the products traded in the two geographic markets compared should be the same or, at least, sufficiently similar.

4.3. Comparison with data from other product markets

Similar to the comparison across geographic markets is an approach used to look at a different product market with similar market characteristics. In particular, the comparator product should be carefully chosen with a view to the nature of the products compared, the way they are traded and the characteristics of the market e.g. in terms of number of competitors, their cost structure and the buying power of customers.

4.4. Combining comparisons over time and across markets

This approach is sometimes called the 'difference in differences' method because it looks at the development of the relevant economic variables in the infringement market during a certain period (difference over time on the infringement market) and compare it to the development of the same variables during the same time period on an unaffected comparator market (difference over time on the non-infringement market) [4].

4.5. Simulation models

Simulation methods draw on economic models of market behavior. Economic studies into how markets function and how firms compete with each other have shown that markets with certain characteristics may allow the likely outcomes of market interaction to be predicted, for instance the likely price or production levels or profit margins [4].

4.6. Cost-based and finance-based methods

The cost-based method consists in using some measure of production costs per unit, and adding a the mark-up for a profit that would have been ‘reasonable’ in the non-infringement scenario [4]. The resulting estimate for a per unit non-infringement price can be compared to the per unit price actually charged by the infringing undertaking(s) to obtain an estimate of the overcharge.

4.7. Other methods

There are also other methods not listed above that can also be used to calculate potential loss [4]. Sometimes less sophisticated tools should be used e.g. companies’ business plans can be taken into consideration.

5. The subject of analysis

In this paper 57 bidding procedures were analyzed. They have been organized under the restrictions of Polish public procurement code in order to choose contractors of building works. All of them have concerned the same type of building object with near the same complexity level and varied value. The data has been standardized for the purpose of calculations to the values from 0 to 1. Table 1 contains an extract from the data collected.

Table 1

Data collected from real bidding procedures

Procedure No.	Bid value		Client’s estimation	No. of bids	Lowest bid value
	Max	Average			
1	0.9509	0.6678	0.7167	7	0.5193
2	0.4097	0.3751	0.6492	7	0.3212
3	0.4329	0.4008	0.6084	9	0.3536
4	0.7877	0.7428	0.5697	3	0.7098

Procedure No.	Bid value		Client's estimation	No. of bids	Lowest bid value
	Max	Average			
6	0.3248	0.2984	0.5400	8	0.2625
7	0.5722	0.5462	0.5046	4	0.5171
8	0.2665	0.2414	0.4734	7	0.2239
9	0.4694	0.4159	0.4703	10	0.3852
10	0.3804	0.2998	0.4566	9	0.2723
11	0.7391	0.4136	0.4536	7	0.3202
12	0.5786	0.5306	0.4432	7	0.4527
13	0.2324	0.2109	0.4365	6	0.1851
14	0.2881	0.2278	0.4077	7	0.1933
15	0.2806	0.2760	0.4020	3	0.2695
16	0.1395	0.1106	0.2658	11	0.0845
17	0.2912	0.1847	0.2595	11	0.1406
18	0.3578	0.2760	0.2561	14	0.2113
19	0.1855	0.1525	0.2502	10	0.1376
20	0.1874	0.1718	0.2055	6	0.1530
21	0.2953	0.2551	0.2025	10	0.2350
22	0.0794	0.0746	0.1752	5	0.0698
23	0.1940	0.1655	0.1436	9	0.1386
24	0.0422	0.0369	0.0504	17	0.0301
25	0.0358	0.0279	0.0436	12	0.0250
26	0.2312	0.2190	n.a.*	5	0.2118
27	0.3448	0.2268	n.a.*	8	0.1920
28	0.0300	0.0229	n.a.*	11	0.0197
29	0.5702	0.4956	n.a.*	8	0.4388
30	0.3501	0.3069	n.a.*	5	0.2731
31	0.0954	0.0876	n.a.*	8	0.0788
32	0.3416	0.2109	n.a.*	7	0.1629
33	0.0294	0.0244	n.a.*	7	0.0208

Procedure No.	Bid value		Client's estimation	No. of bids	Lowest bid value
	Max	Average			
35	0.0575	0.0451	n.a.*	7	0.0390
36	0.3154	0.2695	n.a.*	5	0.2157
37	0.3720	0.3083	n.a.*	12	0.2791
38	0.5221	0.4246	n.a.*	9	0.3663
39	0.6536	0.4946	n.a.*	7	0.3745
40	0.1694	0.1573	n.a.*	3	0.1450
41	0.1158	0.0994	n.a.*	3	0.0873
42	0.0842	0.0745	n.a.*	7	0.0661
43	0.2799	0.2247	n.a.*	12	0.2053
44	0.2669	0.2329	n.a.*	10	0.2177
45	0.6183	0.4363	n.a.*	11	0.3911
46	0.2657	0.2158	n.a.*	12	0.1892
47	0.1312	0.1098	n.a.*	12	0.0944
48	0.0807	0.0698	n.a.*	6	0.0630
49	0.5525	0.5154	n.a.*	6	0.4607
50	0.1851	0.1641	n.a.*	6	0.1540
51	0.3510	0.3091	n.a.*	3	0.2646
52	0.4079	0.3329	n.a.*	10	0.2646
53	0.0887	0.0675	n.a.*	5	0.0547
54	0.1887	0.1588	n.a.*	13	0.1343
55	0.1137	0.0946	n.a.*	9	0.0820
56	0.3546	0.2587	n.a.*	14	0.2236
57	0.1696	0.1479	n.a.*	11	0.1268

* not available

5.1. The analysis of the possibility of collusion with the use of official recommendations

The publication of UOKiK [1] gives some ideas concerning when collusion can be suspected. One of the factor is when there is unusually limited number of bidders. The average number of participants in a bidding procedures listed above was 8.175.

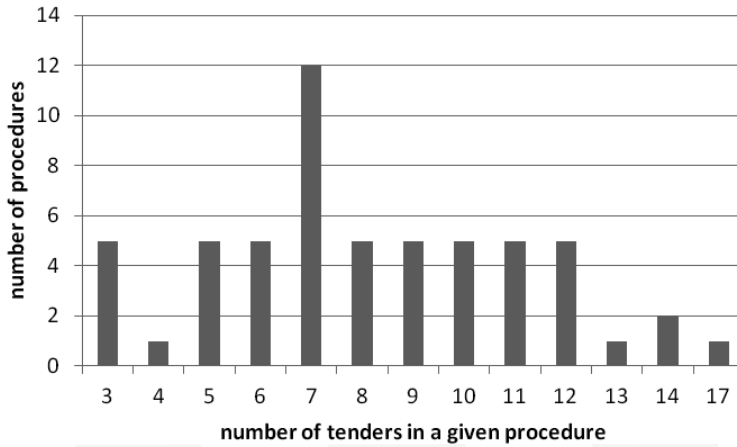


Fig. 1. Histogram of the number of procedures with the number of bidders given on the horizontal axis

In most cases there were 7 bidders for a given building contract. In 52 out of 57 cases (over 91%) there were 4 and more bidders. Another factor for suspecting a collusion is a narrow range of bid values. The range R_i , showing the percentage by which the lowest bid is lower than an average bid in a procedure i , can be calculated using the formula (1):

$$R_i = \frac{(V_{avi} - V_{mini})}{V_{avi}} * 100\% \quad (1)$$

where:

- R_i – the range (defined above) for procedure i ,
- V_{mini} – the lowest value of an offer in procedure i ,
- V_{avi} – the mean average value of offers in procedure i .

The Fig. 2 shows how many bidding procedures have had the range R given within the specified percentage intervals.

It is clearly shown that only within three bidding procedures, bid values have been so close – the range R_i below 5%. The mean average range R_{av} for all analyzed procedures was 13,2 %. Let's indicate the 5 most suspicious bidding procedures from each category. It will be procedure No. 4, 15, 40, 41, 51 (each of them had only 3 bidders) and procedure No. 4, 7, 15, 26, 50 (according to the very low range R_i). Procedures No. 4 and No. 15 fulfill both criteria and the probability of collusion is – in accordance with the criteria described above – the highest. Suppose, that in these procedures, collusion of bidders took place. One of the ways to evaluate a clients loss in the case of collusion, is to calculate the difference between the possible lowest bid value (V_{Mi}) in a procedure i assuming the absence of collusion and the real lowest bid value (V_{mini}) observed in a procedure i . It is important to state here that the bid value was the only criterion for choosing the winner bid in every analyzed procedure. Applying a mean average range from all analyzed procedures instead of a range from a given procedure and mean average value of bids from a procedure i , the formula (1) can be inverted to the formula (2), allowing for the calculation of a possible lowest bid value in a procedure i in the absence of collusion:

$$V_{Mi} = V_{avi} = \frac{(R_{av} - V_{avi})}{100\%} \quad (2)$$

Then, the potential loss on procedure I (marked here as L_i), can be calculated as:

$$L_i = B_{Mi} - V_{\min i} \quad (3)$$

and can be given as a percentage of the real lowest bid observed in procedure i , in order to compare procedures between each other regardless of the monetary value of the contracts, which were concerned.

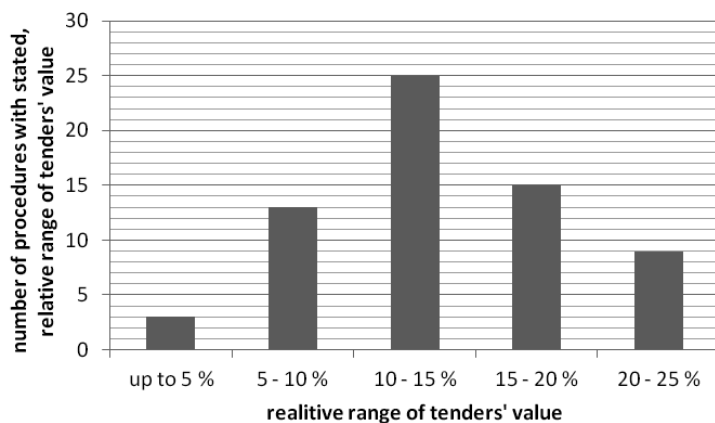


Fig. 2. Histogram of the number of bidding procedures with the value of R within specified percentage intervals

The results for Procedures No. 4 and 15, achieved using formula (2) and (3), are given in Table 2.

Table 2

Calculated value of the Clients' potential loss for Procedure No. 4 and Procedure No. 15, based on the mean average and the range R

Procedure No.	The real lowest bid value	Possible lowest bid value	The difference i.e. potential loss of the Client. L_i	The difference to the real lowest bid (%)
4	0.7098	0.6447	-0.0651	-9.17
15	0.2695	0.2395	-0.0300	-11.13

Applying a method of estimating potential loss of investor in the case of collusion described above, it seems that the potential losses of the Client for procedure No. 4 and No. 15 are similar – quite close to 10% of the value of the real lowest bid respectively.

5.2. The analysis of the possibility of collusion with the use of artificial neural networks

The question that has arisen: if the artificial neural network can predict the potential loss of investor in case of collusion. The calculation is done basing on freeware software [15] – application on MS Excel. Basic assumptions of MLP neural network Model 1 are as follows:

- number of inputs 3 (maximum bid, mean average of bids, number of bids)
- number of outputs 1 (lowest bid)
- number of hidden layers 1
- hidden layer size 4 neurons
- validation set 10% of data, randomly selected
- activation function logistic function

The number of neurons in the hidden layer can be too low and will not allow the network for high quality predictions [13]. The size of the hidden layer seems lower than the optimum one [12] in this application, but it was limited to 4 just to meet the empiric rule stating that the size of hidden layer should be a small part of number of teaching data sets [13]. Researchers haven't fixed the value of this ratio. It depends on the size of the teaching sample and on the range of values of data in the teaching samples [13]. For the Model 1 – described above – the ratio is equal to 4/57. For Model 2 – described below – 4/25. The set of teaching data are comprised of procedures for three years into the future. Most of them are a commercial secret (it is the reason of standardizing them, not quoting). Getting them completed and extending the range of time would allow for increasing the number of teaching samples, and increasing the number of neurons in the hidden layer, and eventually increasing the predictions' precision. Unfortunately the data shown in this paper were all that was available for the authors.

Model 2 differs from Model 1 with the set of inputs. Instead of a mean average bid price, the Clients' estimation is used as an input. It has caused the change in number of learning sets (the Clients' estimations were available for less than 50 % of cases). The number of epochs for error back propagation was set experimentally on 50 for Model 1 and on 100 epochs for Model 2.

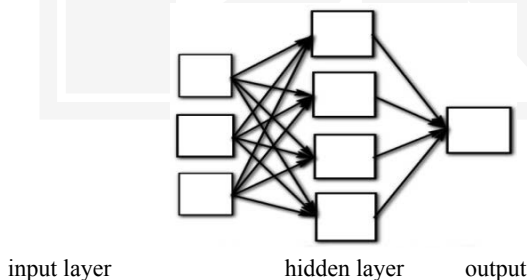


Fig. 3. The scheme of Model 1 and 2 of MLP neural networks used for predictions

The reason for building Model 2 which includes the Clients' estimation is that the Clients' calculation, in a given procedure, compared to values of bids (in the same procedure) is a kind of economic factor. During the prosperity period the Clients' calculations are underestimated (bids are higher than the Clients' expectations), contrary to the crisis period when bids are usually below the expected value. The Fig. 4 shows (on the basis of data from 25 procedures) the ratio of the difference between the Clients' estimation minus the lowest bid, to the Clients' estimation.

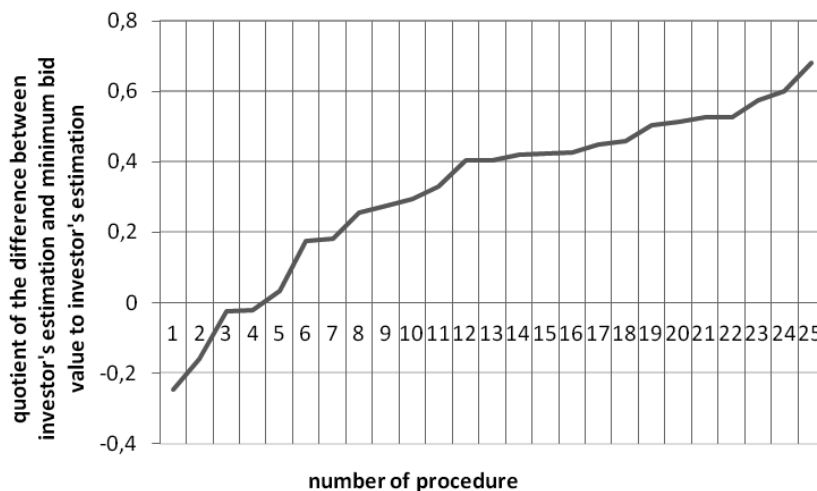


Fig. 4. The ratio of the difference between the Clients' estimation minus minimum bid price, to the Clients' estimation in 25 bidding procedures

The data with the minimum bid price for Procedures No. 4 and no 15 form two models of MLP neural network. Each of these two models were analyzed with two different variants: variant A – data from procedures No. 4 and No. 15 were included in a whole set; variant B – data from procedures No. 4 and No. 15 were excluded from the set. The projections of the potential lowest bids prices in the absence of collusion for aforementioned procedures are shown in Table 3.

Model 1 and Model 2 in both variants shows the possible loss to the Client in the case of assumed collusion in procedure No. 4 (from 29.25% up to 40.77% of the lowest bid price). The same is the case of Procedure No. 15, but only in Model 1A and 1B. The Model 2 (applied for Procedure No. 15) shows (in both variants A and B), that the potential lowest bid is even higher than the observed real lowest bid: there wouldn't be any possible loss of the Client in the case of collusion in the Procedure No. 15. In order to make a decision if the Model 2 is worthwhile, the following considerations should be taken into account:

- whether each of the Clients prepares estimations each time based on the same set of historic data,
- whether Clients' estimations are always prepared based on the historical data (unlike the market price, it is created by the lowest bid price, which is always current),
- how fast the market has changed between the date of Client's estimation and the date of placing bids.

However, the relationship between the economical rating of the building market and the value of bids can be proved, using the relation of Clients' estimation to the lowest bid price as an economic factor, it is not a precise measure according to the three aforementioned matters. Finally, the usage of Clients' estimation value has affected predictions of the possible lowest bid prices giving results much different from the Model 1 and from the methods described in point 5.1 (summarized in Table 2). Moreover Model 2 has only 25 (in variant A) or 23 (in variant B) sets of input data. This makes the MLP neural network a much less precise tool. Moreover the data from two Procedures No. 4 and No. 15 – with a collusion suspected – influence the results of prediction more than twice as big in Model 1.

The comparison of MLP neural network predictions of the Clients' potential loss achieved from the Model 1 (variant A and B), the Model 2 (variant A and B) for Procedure No. 4 and Procedure No. 15

Procedure	Procedure No. 4				Procedure No. 15			
Model	Model 1		Model 2		Model 1		Model 2	
Variant	A	B	A	B	A	B	A	B
Potential lowest bid	0.5011	0.4333	0.5022	0.4204	0.2278	0.2391	0.3032	0.2839
Real lowest bid	0.7098	0.7098	0.7098	0.7098	0.2695	0.2695	0.2695	0.2695
The difference i.e. the potential loss of the Client	-0.2087	-0.2765	-0.2076	-0.2894	-0.0417	-0.0304	0.0337	0.0144
The difference to the real lowest bid (%)	-29.40	-38.95	-29.25	-40.77	-15.47	-11.28	12.50	5.34

Comparing the results achieved using both methods (first, based on mean average bid price and the range R ; second, based on predicting by MLP neural networks, Model 1), Table 4 can be developed.

Table 4

The comparison of MLP neural network predictions of potential Clients' losses achieved from the Model 1 (variant A and B), to an estimation based on mean average and range R for Procedure No. 4 and Procedure No. 15

Procedure	Procedure No. 4			Procedure No. 15		
Model	Model 1		Mean average and range	Model 1		Mean average and range
Variant	A	B		A	B	
Potential lowest bid	0.5011	0.4333	0.6447	0.2278	0.2391	0.2395
Real lowest bid	0.7098	0.7098	0.7098	0.2695	0.2695	0.2695
The difference i.e. the potential loss of the Client	-0.2087	-0.2765	-0.0651	-0.0417	-0.0304	-0.0300
The difference to the real lowest bid (%)	-29.40	-38.95	-9.17	-15.47	-11.28	-11.13

Similar results were achieved for Procedure No. 15 i.e. 15.47%, 11.28% and 11.13% of potential Client's loss in case of collusion. Especially Model 1, variant A, has given 11% of potential loss as it was calculated by mean average and the range R . There is no such a convergence of results for Procedure No. 4, however all methods show the existence of potential loss of the Client in case of collusion.

6. Conclusions

It was confirmed that the lower the ratio (number of neurons in a hidden layer to number of teaching data sets) the more accurate the prediction of an artificial neural network. Estimating the potential loss of the client in the case of collusion has produced quite close results in Model 1 (Procedure No. 15) to the estimation based on mean average and the range of tenders' value concept. According to the fact that results were achieved based only on the assumption that there were collusions in Procedure No. 4 and Procedure No. 15, it cannot be stated which method is more accurate. The existence of a collusion and precision of the aforementioned estimations can be verified based only on a judge's sentence. It is still unknown what was in real life. Were there collusions? What was the size of a loss? The EU Commission recommends calculating the size of a loss based on comparison of unit prices, possible for building construction contracts. It is such a long and drawn out process that it is often not possible to carry out. In that case, utilizing artificial neural networks can be helpful, especially for collusion detecting.

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