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TEMPERATURE EVOLUTION OF THERMODYNAMIC FUNCTIONS FROM SYMMETRIC TO ASYMMETRIC NUCLEAR MATTER

TEMPERATUROWA EWOLUCJA FUNKCJI TERMODYNAMICZNYCH OD SYMETRYCZNEJ DO ASYMETRYCZNEJ MATERII JĄDROWEJ

A b s t r a c t

This paper investigates thermal properties of nuclear matter using the Friedman-Pandharipande-Ravenhall equation of state. Thermodynamic quantities such as internal energy, entropy and free energy are calculated both for symmetric and asymmetric nuclear matter for temperatures ranging up to 30 MeV. A change of free energy curvature indicates the liquid-gas phase transition in nuclear matter.

Keywords: strongly asymmetric nuclear matter, proton localization, equation of state

S t r e s z c z e n i e

Dla równania stanu Friedmana-Pandharipande-Ravenhalla zbadano własności termiczne materii jądrowej. Dla symetrycznej i asymetrycznej materii jądrowej wyznaczono wielkości termodynamiczne, takie jak energię wewnętrzną, entropię i energię swobodną dla temperatur do 30 MeV. Zmiana krzywizny energii swobodnej wskazuje na przejście fazowe ciecz–gaz w materii jądrowej.

Słowa kluczowe: silnie asymetryczna materia jądrowa, lokalizacja protonu, równanie stanu

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1. Introduction

Thermodynamic properties of nuclear matter play an important role in studies of high-energy astrophysical phenomena. The nuclear equation of state at zero temperature governs the structure of cold neutron stars, whereas the equation of state for finite temperatures is necessary for studies of many processes e.g. core collapse of supernovae, black hole formation and neutron star cooling, to name but a few. Knowledge of thermodynamic quantities is required when considering an extremely wide range of densities, temperatures and proton fractions. In order to better understand the properties of dense nuclear matter changes of thermodynamic functions such as internal energy, entropy and free energy with densities, temperatures and degree of asymmetry were evaluated.

This paper is organized as follows. In Section 2, there is a discussion on some of the features of the Friedman-Pandharipande-Ravenhall (FPR) equation of state (EOS) and explicit expressions for thermodynamic functions are given, which are presented in Section 3 for symmetric and asymmetric nuclear matter.

2. Thermodynamics of dense matter with the Friedman-Pandharipande-Ravenhall equation of state

The EOS of dense nuclear matter is an essential ingredient in modelling neutron stars. Knowledge of the EOS, particularly with arbitrary isospin asymmetry, i.e. different proton and neutron fractions, is of fundamental importance for both nuclear physics and astrophysics. The saturation density and the energy per particle of nuclear matter can be used to test properties of finite nuclear systems extrapolated to the thermodynamic limit. Moreover, the study of the EOS of asymmetric matter allows us to shed some light on the behavior of the isospin asymmetry energy. In our approach, we use the FPR model in which the density of energy as a function of neutron n_N and proton n_P densities reads [1]:

$$\varepsilon\left(n_N, n_P\right) = \left(\frac{1}{2m_N} + B_N\right)\tau_N + \left(\frac{1}{2m_P} + B_P\right)\tau_P + n_B^2\left(a_1 + a_2 e^{-b_1 n_B} + \left(\frac{1}{2} - x\right)^2\left(a_1 + a_2 e^{-b_1 n_B}\right)\right) + n_B e^{-b_1 n_B^2}\left(a_5 + a_6 n_B + \left(\frac{1}{2} - x\right)^2\left(a_7 + a_8 n_B\right)\right), \tag{1}$$

where $n_B = n_N + n_P,\; x = \dfrac{n_P}{n_B}$,

$$B_i = \left(a_9 n_B + a_{10} n_i\right) e^{-b_3 n_B},$$

$$\tau_i = \frac{3}{5}\left(3\pi^2\right)^{\frac{2}{3}} n_i^{\frac{5}{3}} \quad i = N, P.$$

The effective proton mass is as follows:

$$\frac{1}{2m_P^*} = \frac{1}{2m_P} + a_9 n_B e^{-b_3 n_N}. \tag{2}$$

The parameters in (1) are: a_1 = 1054.0 MeV fm^3; a_2 = –1393.0 MeV fm^3; a_3 = –2316.0 MeV fm^3; a_4 = 2859.0 MeV fm^3; a_5 = –1.78; a_6 = –52.0 MeV fm^3; a_7 = 5.5 MeV; a_8 = 197.0 MeV fm^3; a_9 = 89.8 MeV fm^5; a_{10} = –59.0 MeV fm^5; b_1 = 0.284 fm^3; b_2 = 42.25 fm^6; b_3 = 0.457 fm^3.

A major advantage of the above effective interactions is that they can be used straightforwardly to make finite-temperature calculations. In our approach, it is assumed that the kinetic energy densities and baryon matter densities are the only quantities that exhibit dependence on temperature [2]:

$$\tau_i = \frac{2}{(2\pi)^2}\left(2m_i^* T\right)^{\frac{5}{2}} J_{\frac{3}{2}}\left(\eta_i\right), \qquad i = N, P, \tag{3}$$

where $\eta_i = \frac{\mu_i}{kT}$ (μ_i the chemical potential of neutrons or protons) can be derived from the baryon number density:

$$n_i = \frac{2}{(2\pi)^2}\left(2m_i^* T\right)^{\frac{3}{2}} J_{\frac{1}{2}}\left(\eta_i\right). \tag{4}$$

Eqs. (3) and (4) are written in terms of Fermi integrals:

$$J_\nu\left(\eta\right) = \int_0^\infty dx \frac{x^\nu}{1+\mathrm{e}^{x-\eta}}. \tag{5}$$

Calculated entropy per baryon is:

$$S_i = \frac{5}{3}\frac{1}{n_i}\frac{1}{4\pi^2}\left(2m_i^* T\right)^{\frac{3}{2}} J_{\frac{3}{2}}\left(\eta_i\right) - \frac{1}{2}\eta_i, \tag{6}$$

where:

m_i^* – the effective nucleon masses of neutrons and protons.

While studying the thermodynamics of dense matter, it is convenient to choose the Helmholtz free energy. From Eqs. (1) and (6), the expression for the free energy per baryon reads:

$$F = (\varepsilon\left(n_N, n_P, T\right) - T\left(n_N S_N + n_P S_P\right))/n_B, \tag{7}$$

where the energy density $\varepsilon(n_N, n_p, T)$ depends on temperature. In the next section, we discuss the properties of internal energy, entropy and free energy in the FPR model.

3. Properties of symmetric and asymmetric nuclear matter at finite temperatures

At finite temperatures, the nuclear matter structure and properties are not as well settled as at zero temperature. In this paper, the thermodynamic properties of dense matter for the FPR equation of state are discussed. To make the discussion complete, the energy per particle, the entropy per particle and the free energy per particle are computed. Based on the calculated free energy, all other thermodynamic quantities may be obtained from standard thermodynamic relations.

In Fig. 1, the FPR equation of state for symmetric nuclear matter (proton fraction $x = 0.50$) are displayed for different temperatures $T = 0$, 10, 20 and 30 MeV.

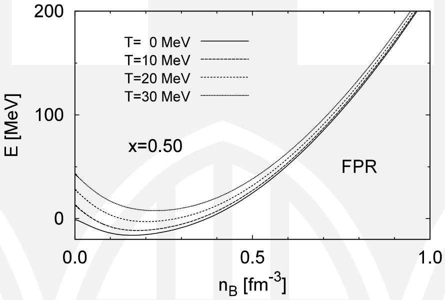

Fig. 1. Energy per nucleon versus baryon density for symmetric nuclear matter at different temperatures

For very small densities (below 0.02 fm^{-3}), one obtains the behavior of a free Fermi gas with linear temperature dependence and for increasing density, quadratic temperature dependence [3]. The entropy per baryon and the free energy per baryon for symmetric matter are shown in Fig. 2 and Fig. 3 for different temperatures. For symmetric nuclear matter, the entropy behaviour (Fig. 2) agrees very closely with the experimental results [4].

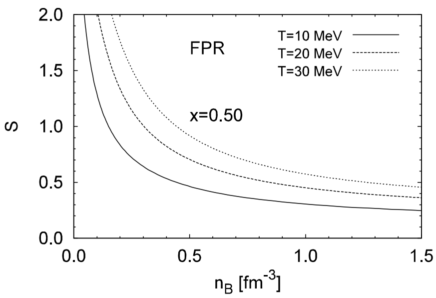

Fig. 2. Entropy per nucleon versus baryon density for symmetric nuclear matter at different temperatures

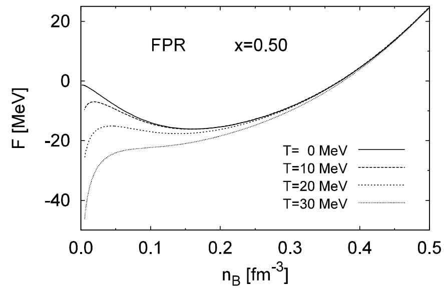

Fig. 3. Free energy per nucleon versus baryon density for symmetric nuclear matter at different temperatures

In Fig. 3, the free energy per nucleon versus baryon density exhibits an unstable region of negative curvature for lower densities below $T = 20$ MeV (cf [5]). Non-convexity of $F(n_B,T)$ with respect to n_B at a fixed temperature implies a negative isothermal compressibility K_T which violates the stability relation $K_T > 0$. In this region, the physical equation of state can be obtained by performing the Maxwell construction. This signifies the presence of a first-order phase transition. The existence of a critical temperature for nuclear matter is extremely strong evidence that, under appropriate conditions, there should be a transition between a nuclear 'liquid' and nuclear 'gas'. The physics of nuclear matter is therefore a crossover from a gas of nucleons to homogeneous matter, where nuclei and larger clusters coexist with the nucleon gas over a wide range of intermediate densities. At temperatures ($T \sim 20$ MeV) and high densities, a liquid-gas type of phase transition was first predicted theoretically by A.L. Goodman [6] and later observed experimentally in a nuclear multi-fragmentation phenomenon [7].

The case of asymmetric matter is more complex to study since there is an additional degree of freedom to consider – the isospin asymmetry, i.e. different neutron and proton fractions. Such matter plays an essential role in astrophysics, where neutron-rich systems are involved in neutron stars and type-II supernovae evolution [8].

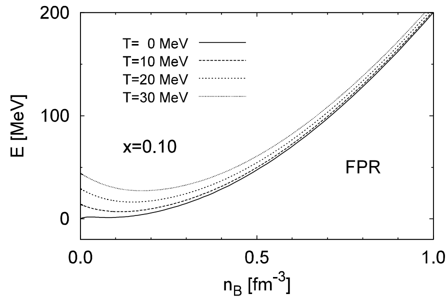

Fig. 4. Energy per nucleon versus baryon density for asymmetric nuclear matter ($x = 0.10$) at different temperatures

Fig. 5a presents the entropy density S versus baryon density for asymmetric matter (x = 0.10). This quantity is the sum of two contributions – the first from neutron entropy (Fig. 5b), and the second from proton entropy (Fig. 5c). In asymmetric nuclear matter, the contribution of S_P to total entropy is much greater than S_N – as follows clearly from relation (7).

The dependence of free energy of asymmetric nuclear matter versus baryon density (Fig. 6) shows non-convexity at temperatures below T = 10 MeV for proton fraction equaling x = 0.10. It indicates that in asymmetric nuclear matter, the phase transition occurs in much lower temperatures than in symmetric one.

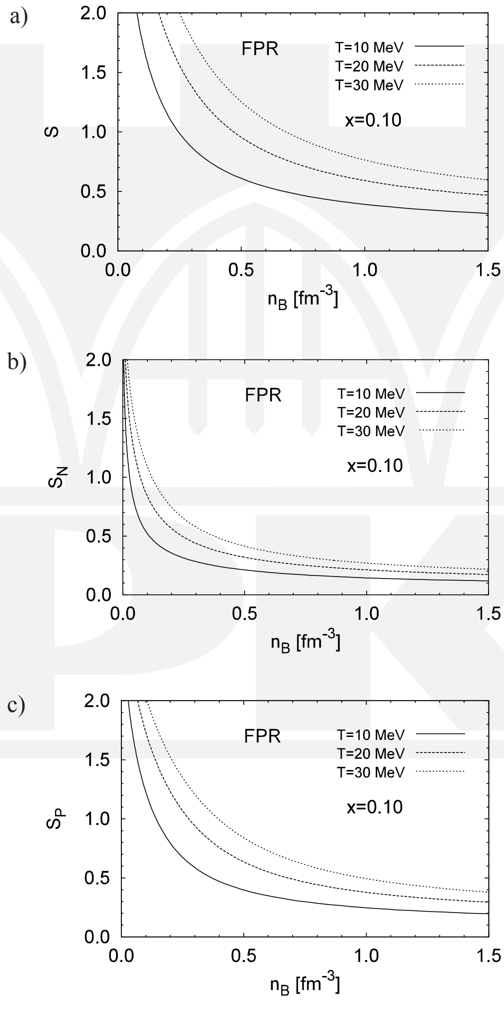

Fig. 5 a) Entropy per nucleon versus baryon density for asymmetric nuclear matter (x = 0.10) at different temperatures; b) neutron entropy contribution; c) proton entropy contribution

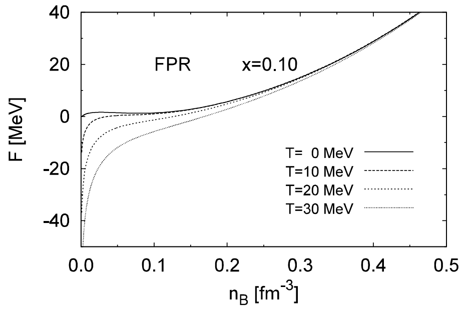

Fig. 6. Free energy per nucleon versus baryon density for asymmetric nuclear matter ($x = 0.10$) at different temperatures

In Fig. 7, changes of the free energy with densities at constant temperature $T = 10$ MeV for different asymmetry of nuclear matter are also shown. It was observed that phase transition in asymmetric nuclear matter takes place for lower densities than in symmetric nuclear matter.

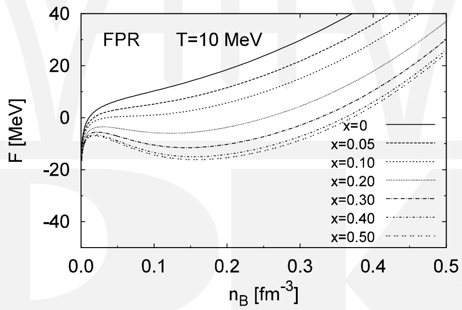

Fig. 7. Free energy per nucleon versus baryon density at $T = 10$ MeV from fully asymmetric ($x = 0$) to symmetric ($x = 0.50$) nuclear matter

4. Conclusions

The thermodynamic properties of hot, dense nuclear matter employing the Friedman-Pandharipande-Ravenhall model of nuclear interaction were investigated. Evidence of the liquid-gas phase transition both in symmetric and asymmetric nuclear matter was observed. As this paper shows the critical temperature strongly decreases with nuclear matter asymmetry. The results presented here lead to better understanding of phase diagram of nuclear matter.

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