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Hyperelastic behaviour of auxetic material in tension and compression tests

Hipersprężystość materiałów auksetycznych na przykładzie testu jednoosiowego rozciągania i ściskania

Abstract

This paper presents the numerical simulation of uniaxial tension and compression tests for negative Poisson's ratio materials subjected to large strains. Numerical calculations are performed for the determination of the material characteristics of auxetic periodic lattices. The finite element method (FEM) coupled with 2D periodic homogenisation technique is used. The results show the existence of large variations in strain-stress plots, which can be achieved by changing the lattice geometry parameters.

Keywords: auxetic microstructure, hyperelasticity, material characteristics

Streszczenie

W artykule przedstawiono numeryczne symulacje testów jednoosiowego rozciągania i ściskania dla materiałów o ujemnym współczynniku Poissona w zakresie dużych odkształceń. Celem wyznaczenia charakterystyk materiałowych wykonano obliczenia numeryczne dla materiałów auksetycznych o strukturze periodycznej. Zastosowano metodę elementów skończonych połączoną z teorią homogenizacji. Wyniki wskazują na dużą różnorodność otrzymanych ścieżek naprężenie-odkształcenie uzyskanych przez zmianę parametrów geometrycznych struktury materiału.

Słowa kluczowe: mikrostruktura auksetyczna, hipersprężystość, charakterystyka materiałowa

1. Introduction

Materials with cellular microstructure are used in a variety of structural applications due to the enhancements they offer with regard to many mechanical properties. Contrary to ordinary materials, in which work is restricted to a linear elastic regime, cellulars reveal nonlinear elastic behaviour and deformability in the range of large strains. Among cellular microstructures are special lattices which produce negative Poisson's ratio materials, these are called auxetics. Such materials reveal counterintuitive behaviour. These materials are of particular interest due to their improved properties such as fracture toughness, shear resistance, indentation resistance, elastic energy absorption and energy damping. Auxetics possess a wide range of applications in innovative smart structures and composite materials. They are used in the aerospace and automotive industries.

The development of the concept of auxeticity dates from the first publication by Lakes in 1987 and the fabrication of micromorphous polethylene with a negative Poissons ratio. In 1988, Gibson realised the auxetic effect for silicone rubber and aluminium honeycombs. In 2004, Yang formulated the basis for molecular design within the field of nanotechnology. Great contribution to the development of auxetic materials was made by Alderson [1], who manufactured, tested and found potential applications of cellular solids, polymers and composites. A recent extensive review on the properties of auxetic materials compared with positive ratio materials was provided in an article published in 2011 in Nature Materials [8] by Greaves, Greer, Lakes & Rouxel.

For cellular materials, a variety of microstructures were developed to achieve auxetic behaviour. A re-entrant honeycomb structure was first used by Gibson and Kolpakov. The auxetic effect in foam and re-entrant honeycombs lies in the unfolding of re-entrant cells when they are stretched. Symmetries represented by non-chiral structures were proposed by Lakes, Theocaris, Smith and Gaspar. Chiral microstructures were proposed by Prall, Lakes and Grima. In chiral structures, the auxetic effect is achieved through the unwrapping of the ligaments around the circular nodes.

Cellulars reveal strongly nonlinear behaviour within the elastic range. A micromechanics framework for the development of continuum-level constitutive models for the large-strain deformation of porous isotropic hyper-elastic materials is given by Danielsson, Parks & Boyce [3], and Horgan [10] for non-compressible materials. Murphy formulated a strain energy function for nonlinear compressible isotropic materials. An alternative approach was introduced by Ogden [9] who modelled compressible materials by using strain energy functions based on polynomial functions of the principal stretches. Anisotropic hyper-elastic materials form a narrow group among constitutive models which describe elastic behaviour. An energetic approach to the analysis of anisotropic hyper-elastic materials is proposed by Vegori et al., [18]. According to work by Dłużewski [6], the explicit form of a constitutive equation for anisotropic nonlinear materials for Cauchy stress has not yet been formulated. When the material undergoes finite deformation, the material properties are strongly dependent on the deformation and the effective material properties have to be calculated for each deformation state (Vegori, [18]). Therefore, it is possible for instantaneous stiffness in uniaxial strain test.

The mechanical properties of cellular solids are controlled by both constituent materials and cell topologies. The cell topologies can function as either load-bearing structures or flexible structures. If a cellular solid is used for a structural purpose, it should be stiff, otherwise, it should be compliant. The design of stiffness is possible through the selection of material and cell topologies.

To obtain effective properties of cellular material, classical homogenisation theory can be used (Nemat & Naser, [16]). The hyper-elastic behaviour of cellular structures at small strain was described by Janus-Michalska [11]. The concept of obtaining effective material properties using a numerical homogenisation procedure under finite deformation is given by Nakshatrala et al. (2013).

In the present paper, effective material characteristics, tangent Young moduli functions and Poisson's ratio functions are calculated for a set of cellular microstructures in tension-compression tests. The influence of geometric microstructural parameters on effective material properties is tested.

2. Micromechanical modelling of auxetic material

Microstructured auxetic material is modelled using a beam structure with the 2D re-entrant lattice shown in Fig. 2a. The properties of an equivalent continuum can be obtained through homogenisation. The classical theory of continuum is sufficient for this microstructured body. The following procedure given by Nemat-Naser's [16] with representative unit cell of the geometric parameters shown in Fig. 1b is studied.

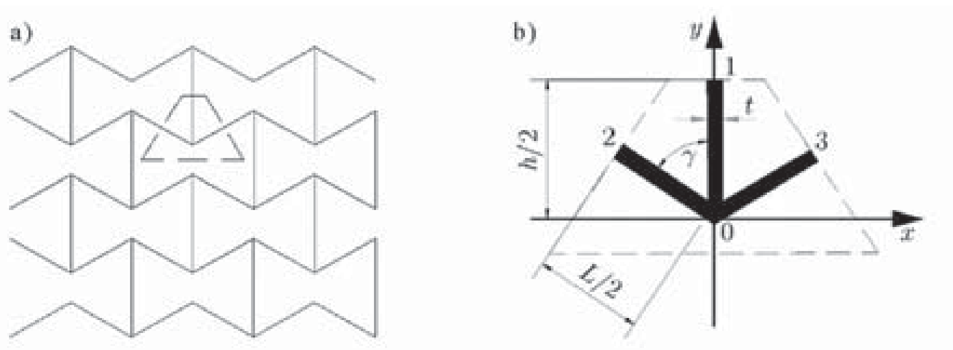


Fig. 1. a) Auxetic structure; b) representative unit cell

Analytical homogenisation was used for cellular materials with typical symmetries by Janus-Michalska [12, 13]. For auxetic structures, numerical homogenisation was used to obtain the material constants for small strain regime [11].

The deformation of the cell model can then be solved numerically as a boundary value problem and the macroscopic stress-strain response can be extracted. Details on the cell model, boundary conditions and the calculation of the macroscopic properties and framework are described in works [11–13].

3. Strain dependent Poisson's function

The Poisson's ratio is defined for infinitesimal strain. The measure was first introduced by Simeon Dennis Poisson (1787–1840). The definition is as follows: $\nu_{12} = {}^{-\varepsilon_2}\!/_{\varepsilon_1}$

where: 1 – denotes the direction of stretching, 2 – denotes the perpendicular direction.

For highly nonlinear elastic materials, the definition is extended to a strain dependent Poisson's function. This is analogous to the definition of a tangent Young modulus. For auxetic materials, Poisson's ratio is usually highly strain dependent, moreover, the Poison's ratio can be negative only over the certain strain range.

4. Parametric study

Materials of several geometric configurations are considered. The geometric parameters are collected in Table 1, the structures are shown in Fig. 2.

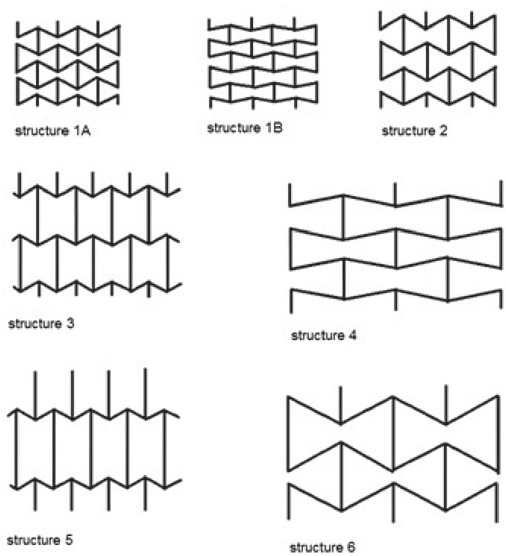


Fig. 2. Geometrical configurations of auxetic structures

Table 1. Microstructural geometric parameters

Structure	L [mm]	H [mm]	γ [°]	t [mm]
1A	1.50	1.65	60	0.15
1B	1.50	1.50	80	0.15
2	1.50	2.00	60	0.15
3	1.50	3.00	80	0.15
4	3.00	1.40	80	0.15
5	1.50	3.00	60	0.15
6	3.00	4.00	60	0.15

Material skeleton with the following material data: $E_s=10GPa$, $\nu_s=0{,}3$, ${}^{S}R_m=10MPa$ is adopted, where: E_s, ν_s -elastic constants, ${}^{S}R_m$- rupture modulus.

Computations were carried out in the ABAQUS system with the use of Timoshenko beam elements.

5. Results

Tension-compression tests were carried out for each microstructured material in the X and Y directions. Material characteristics are given in Fig. 3 and Fig. 4. Stress-strain plots ends for rupture of skeleton material for tension and contact of skeleton beams for compression.

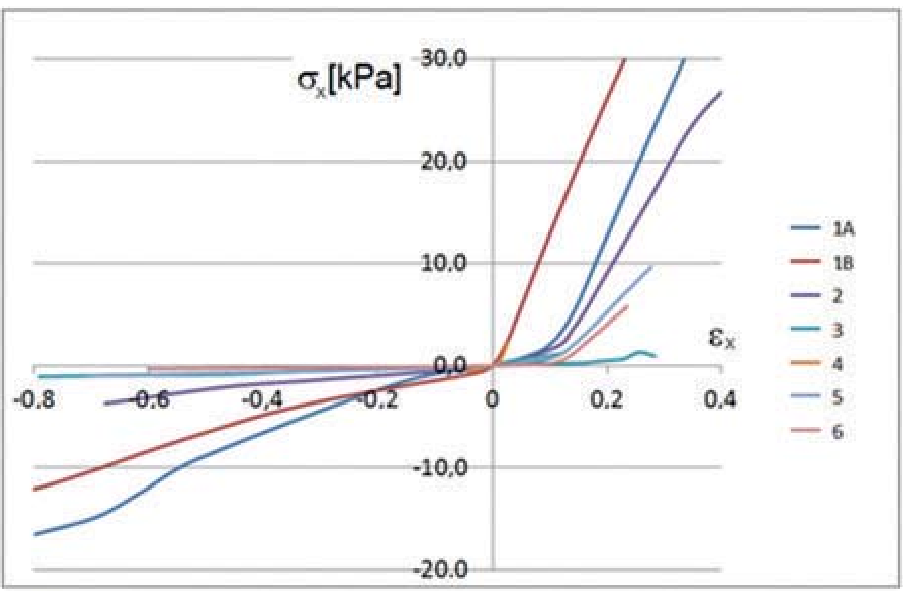


Fig. 3. Material characteristics for tension-compression in the X direction

It can be observed that in the test in the X direction, the stiffness decreases under compression and increases under extension. This results from microstructural deformation effects. The increasing of equivalent material stiffness is related to the extension of beams in the microstructure, which is governed by tensional stiffness. The decreasing of the equivalent material stiffness is related to the deflection of the beams in the microstructure – this is governed by flexural stiffness. The relationship between stiffness and the H, L and γ parameters is clearly visible. For increasing parameters, the tensional stiffness decreases, whereas the compression stiffness increases.

In the test in the X direction, the stiffness increases under compression and decreases under extension. The relation between of stiffness and he H, L and γ is not clearly visible.

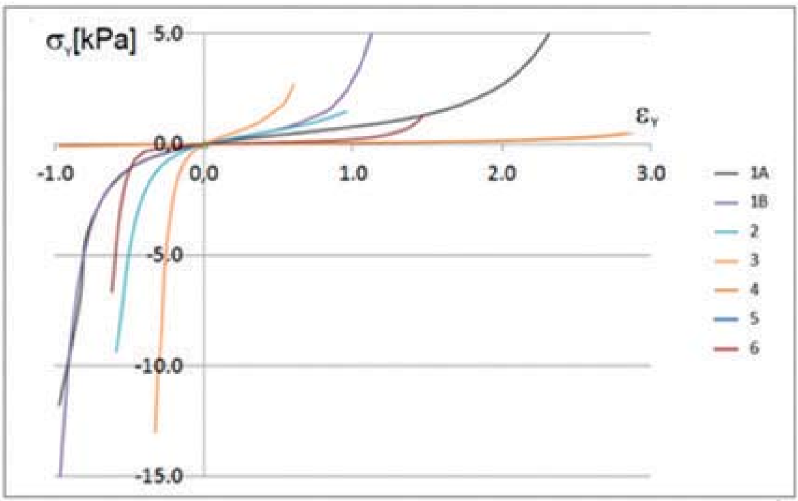


Fig. 4. Material characteristics for tension-compression in the Y direction

A variety of equivalent material properties calculated for infinitesimal strain state can be obtained due to changes of geometrical parameters and skeleton material constants. For the chosen structures the equivalent Young moduli and Poisson's ratios are collected in Table 2.

Table 2. Equivalent material constants

Structure	E_X [kPa]	E_Y [kPa]	ν_{XY}	ν_{YX}
1A	13.53	3.62	-15.21	-0.481
1B	46.18	1.29	-5.058	-0.142
2	6.27	1.941	-1.718	-0.532
3	20.90	2.808	-2.289	-0.307
4	20.07	0.058	-17.708	-0.053
5	3.67	3.407	-0.9823	-0.912
6	0.783	0.2402	-1.786	-0.547

For large strains, plots of the functions of tangent elastic moduli are presented in Figs 5 and 6.

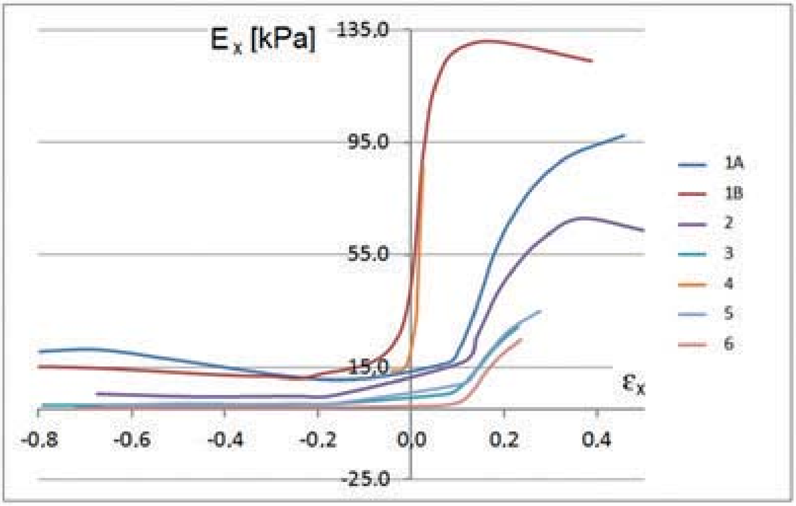


Fig. 5. Plots of elastic modulus E_X for microstructured equivalent continua

The relation between the elastic modulus E_X and the parameters H, L and γ is clear. For increasing parameters, the tangent elastic modulus E_X decreases. The relation between the elastic modulus E_Y and the parameters H, L and γ is not visible.

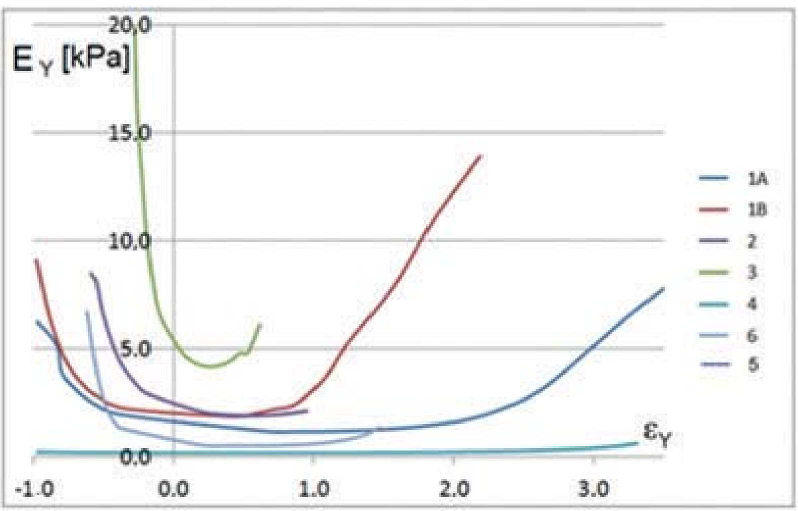
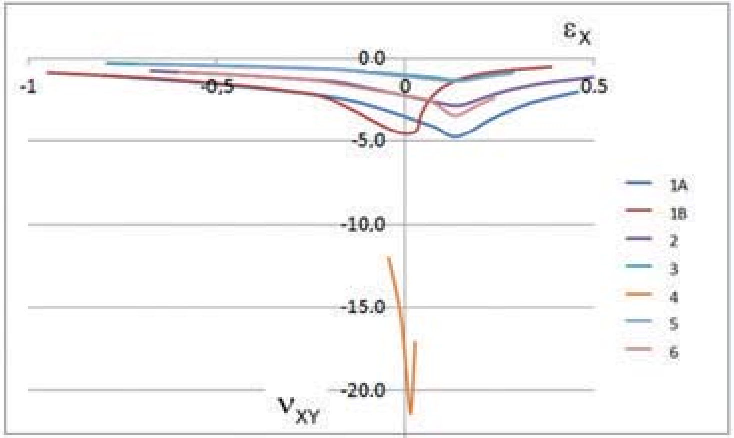


Fig. 6. Plots of elastic modulus E_Y for microstructured equivalent continua

Plots of the functions of Poisson's ratio are presented in Figs 7 a & b and Fig. 8.

a) 

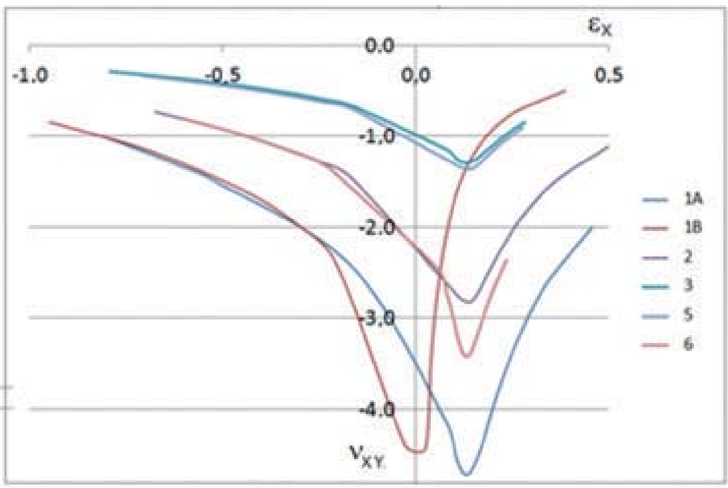
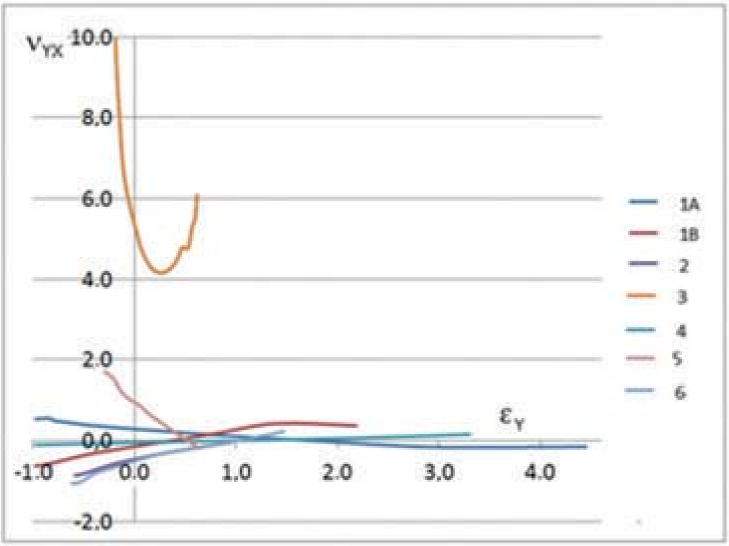
b) 

Fig. 7. Plots of function of elastic Poisson's ratio for the tension-compression tests in the *X* direction

The plot for Structure 4 in Fig. 7a gives exceptionally high values of the Poisson's ratio function with relation to other structures. Other plots are presented in Fig. 7b to precisely show their changeability. It is worth noting that for tension-compression tests in the *X* direction in the whole range of strains, the Poisson's ratio functions remain negative.

For tension-compression tests in the *Y* direction, the plot for Structure 3 in Fig. 7a gives exceptionally high and positive values of Poisson's ratio function. Other plots are presented in Fig. 8b to precisely show their changeability. It is worth noting that Poisson's ratio functions can have negative or positive values.

a)



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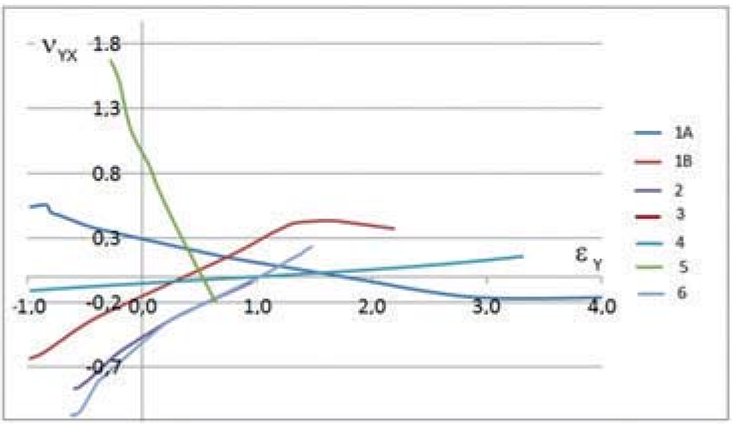

Fig. 8. Plots of function of elastic Poisson's ratio for the tension-compression test in the *X* direction

6. Conclusions

The tested class of microstructered cellulars shows that the behaviour of materials with the auxetic beam microstructure is significantly nonlinear. The paths of material characteristics are dependent on the tension-compression direction and on the material and geometric microstructural parameters. An attempt to assess the type of dependence is made.

Poisson's ratio and tangent stiffness Young moduli functions are introduced to visualise the effect of the influence of large strain deformation; it shows the ability of modelling microstructures to achieve the required properties.

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