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METHODS OF IDENTIFYING THE OPTIMAL POSITIONING OF DIFFERENT PRODUCTS WITHIN A WAREHOUSE WITH REGARD TO MINIMISING THE COSTS ASSOCIATED WITH ORDER PICKING

Abstract
The most important factors conditioning the competitiveness of a warehouse are time and money. A randomly or improperly chosen storage process influences three types of costs: the movement of goods, waiting time, transportation costs. To increase the effectiveness of goods completion, i.e. to decrease processing time and cost, appropriately selected methods of product classification are used. In the article, the authors have done simulations of product classification for an ABC analysis, an ABC analysis together with a COI index, an ABC analysis together with an XYZ analysis, a COI index only, and the method of free product storage places. Simulations were made for two variants taking into account the need and there is no need to shifting products on the completion trolley during the whole completion process.

Keywords: warehousing, warehouse activities, methods of product classification, simulation, performance analysis

Streszczenie
Najważniejszymi czynnikami warunkującymi konkurencyjność magazynu są czas i pieniądze. Przypadkowo lub źle dobrany proces składowania może powodować 3 rodzaje marnotrawstwa: ruchu, oczekiwania i transportu. Aby zwiększyć skuteczność kompletacji produktów, np. w celu zmniejszenia czasu kompletacji i kosztów, należy odpowiednio dobrać metodę klasyfikacji produktów. W artykule autorzy wykonali symulacje klasyfikacji produktów dla analizy ABC, ABC wraz z Index COI, ABC wraz z analizą XYZ, Index COI oraz metody składowania wolnych miejsc. Symulacje zostały wykonane w dwóch wariantach, biorąc pod uwagę konieczność i brak konieczności przemieszczania produktów na wózek kompletacyjnym podczas całego procesu kompletacji produktów.

Słowa kluczowe: magazynowanie, procesy magazynowe, metody klasyfikacji produktów, symulacje, analiza wariantowa
Nomenclature

- \( c(x) \) – the number of routes running across the warehouse to row \( x \)
- \( c_w(y) \) – the number of route running among the warehouse to the rack \( y \)
- \( d_{cr} \) – route width
- \( D_{lr} \) – rack length
- \( D_{wr} \) – shelf space width
- \( p \) – the number of storage levels, wherein \( p = 1, \ldots, N \)
- \( r \) – the coefficient reflecting the rack in the \( W \) matrix assumes the value \( r(x, y) \in \{0, 1\} \), wherein \( r(x, y) = 0 \) for the corridors, for the remaining places \( r(x, y) = 1 \)
- \( t(p) \) – time for lifting and lowering the forks for a given storage level \( p \)
- \( t_d \) – access time to the rack
- \( t_l \) – time needed to overcome the arc (change of direction by at least 90°)
- \( t_{lp} \) – time needed to move a distance of 1 meter in a straight line
- \( t_p(x, y, p) \) – time of moving around the warehouse
- \( x \) – the number of the row number in the warehouse \( x = 1, \ldots, N \)
- \( y \) – the number of the rack in a row \( y = 1, \ldots, N \)

1. Introduction

The warehouse as an element of the supply chain is an important part of activity in the distribution and production of goods and raw materials. Continuously developing trends and the growing pressure resulting from these numerous changes have unavoidably altered the structure of the supply chain and the localisation and operation of warehouses. As a result, this has forced the introduction of stock optimisation strategies which also incorporates the time of order completion, cost minimisation and increases in the level of service delivered to clients.

The global market allows for a competition of local enterprises as well as of those ones who have their localizations on other continents. This is the case because more and more enterprises are deciding to sell their products through the Internet (e-business, e-commerce). This results in smaller and smaller orders and that have irregular frequencies. For this reason, the appropriate design of the warehouse layout and product distribution strategies constitutes a serious challenge to enterprises [1].

The most important factors that influence the competitiveness of warehouses are time and money [2–4]. If the storage process is chosen at random or is improper, then it affects three types of costs which relate to movement, waiting time and transport. This occurs, for example, when high-demand products are stored in the most distant part of the warehouse and those rarely in need are at the closest distance to the completion and packing zone [5–7]. Thus, the time, distance and internal stock-movement costs become higher. The number of employees and stock-movement devices also increases [8].

The performed analyses show that the movement of goods constitutes half of the total order completion time [9, 10]. Its final duration depends, among other things, on the level of warehouse automation, the applied storage system, and the way of order completion. If a given
enterprise wishes to remain competitive, each process which takes place in the warehouse should be analysed and, firstly, the time spent on stock movement needs to be decreased [11]. The optimisation of transport processes in the warehouse allows to naturally reduce the time of completion, and thus the way between the most important products.

Figure 1 shows the percentage share of the individual activities which constitute the process of order execution. The time of completion of the order depends primarily on the number of products from the list of completion, the distance between them, the warehouse and transportation system.

![Figure 1. Percentage share of the individual activities which constitute the process of order processing – on the basis of [12, 13]](image)

Nowadays, product classification methods are used to plan the placement of products within the warehouse. These methods rely on assigning products to groups of various ranks. The products are then located in the warehouse in such a way that the shortest time of access to certain positions which are most significant is provided [14, 15]. Classical methods of product classification are the following analyses: ABC, XYZ, EIQ and COI index [16]. Despite the prevalence of the use of none of the methods does not give the right effect with regard to the requirements of warehouse. The most common reason is to optimize for a single criterion, some optimization is repeated with a different criterion, then the result is according to the results obtained. For this reason, companies are looking for new ways to combine the methods which are currently used.

**METHODS OF PRODUCT CLASSIFICATION**

**The COI index** is the easiest method of product classification [17]. This method is based on a two-criterion analysis in which the size of the product and the demand for it are the criteria. The size of the product is identified in terms of either its volume or its weight and the demand refers to the number of product picking (its popularity) or the average requisition. Through the application of the COI, it is possible to distribute products in such a way that those with the lowest index are closest to the packing zone. Thus, the distance covered by the largest/heaviest products is shortened. The dependence of the size of the product on its demand allows finding the value in between those two criteria [14, 18].
The ABC analysis is the most frequently applied analysis that enables product classification. The standard ABC analysis allows the division of products into three groups with the percentage participation equal to: A – 80%, B – 15%, C – 5%. Some modifications of this analysis are also used. These are comprised of more groups with adequately corrected percentage participation. The ABC analysis is a one-criterion analysis; therefore, it is impossible to take into consideration a few input parameters at the same time. However, it is possible to perform the analysis several times – each time with a different characteristic considered as a criterion. The synthesis of the results is then conducted with taking proper weights for each criterion (the result of the analysis) [19–22].

The XYZ analysis enables the ABC analysis to be complemented with an additional criterion through making classification inside the already isolated groups. The most frequently used criterion in the XYZ analysis is the regularity of usage defined on the basis of the historical sales data. Unlike the ABC analysis in which the classification is made according to the popularity of the products or the amount of items sold, the XYZ analysis enables independent evaluation on the basis of the individual index for each product [23, 24].

2. THE BASICS OF A VARIANT ANALYSIS

To evaluate the effectiveness of the product classification methods, computer simulations were carried out for the ABC analysis, the ABC analysis together with the COI index, the ABC analysis together with the XYZ analysis, the COI index only, and the method of randomly allocated storage locations. This has enabled engineers to decide which criterion taken for the product classification in warehouses has the greatest influence on the shortening completion time. Simulations were conducted for the following variants:

**Variant I** – assumes that there is no need for shifting products on the completion during the process of executing an order.

**Variant II** – assumes that there is a need for shifting products on the completion trolley during the whole completion process. This need is dependent on the weight, product volume and the type of packaging (its response to stacking). If there is a risk of damaging the product which is on the lower level of the trolley by the product on the top, then it is necessary to change the order of the products.

For the development of the product distribution method, the following assumptions were adopted which define the type of storage for which the presented method is appropriate:

- The warehouse structure can be described in the form of a matrix. This enables calculation of the distance between particular picking bays in the warehouse, taking into account the parameters of the warehouse such as width and length of the rack, width of the aisles between the racks, and the number of transverse and longitudinal aisles.

- The warehouse does not use ‘goods to human’ picking, i.e. automatic stacker cranes and conveyors in the product storage area. All orders are completed by employees using forklifts and order pickers that move in the product storage area. The beginning and end of the picking process takes place in the packing area.
The picking process can be supported by any WMS system and automatic identification tools.

Products with significant differences in weight and overall dimensions can be stored in the warehouse.

It is permissible to have separate zones for the storage of dangerous goods, foodstuffs and general stock in the warehouse.

The lifting and lowering time of the pallet is 65 seconds.

The lifting speed of the forks is 20 m/s.

The speed of moving around the warehouse is 12 m/min.

The simulations were conducted on the basis of the 1000 generated lists of the completion of products characterized by the following parameters:
- weight – from 0.1 to 6 kg,
- volume – from 0.1 to 0.4 m³,
- the number of product types on the order list – from 3 to 20 items,
- the number of units of each product – from 1 to 60.

To evaluate the effectiveness of the order fulfilment derived from the applied method of product distribution in the warehouse, one should use a model which allows precise representation of processes occurring in the real warehouse. Therefore, a special method was used – this method shows clients’ orders which undergo the completion process in warehouses, i.e. the method uses random lists of product completion. On the basis of the generated completion lists the route of product completion was determined with the use of a method applied by the majority of medium size enterprises which have their own warehouses. This method determines the completion routes by relying on the closest point with reference to the current location of the warehouseman. The method also concerns the need of interference in the arrangement of products on the completion trolley if there is a risk of crushing a smaller product with a larger item, i.e. sensitivity to stacking. The optimisation target is described by the formula:

\[ PO_{min} : F(t_p) = \sum_{x=1}^{xa} \sum_{y=1}^{ya} \sum_{p=1}^{pa} (d(x, y) + t(p)) \]  

(1)

The warehouse structure is described by the rack – value 1 and empty spaces with value 0 matrix (2):

\[ W = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1y} \\ r_{21} & r_{22} & \cdots & r_{2y} \\ \vdots & \vdots & \ddots & \vdots \\ r_{x1} & r_{x2} & \cdots & r_{xy} \end{bmatrix} \]  

(2)

For calculating the time used for movements during the picking process, formula (3) was used.

\[ t_p(x, y, p) = t_{iq} (D_{iw} \cdot (x - c_i(x)) + d_{iw} \cdot c_i(x)) + t_{iq} (D_{wr} \cdot (y - c_w(y)) + d_{wr} \cdot c_w(y)) + t_1 + t(p) \]  

(3)
Based on the warehouse structure matrix (2) and formula (3), the correction of moving time was performed (4).

\[ W(x, y-1) = 0 \text{ to } t_p = t_p - t_p(D_{wr} + d_{cr}) \]  

(4)

For simulations, algorithms created by the authors were used. The algorithms constitute the basic software written in the PHP language. They use relation mySQL databases [17]. As a result of this solution, it is possible to use great data sets in order to present the results in a clear way, and to easily modify the input parameters. Moreover, this solution enables the integration of the basic software with the Matlab software. The architecture of the developed system is presented in Fig. 2.
3. Variant I

Variant I was created for a large warehouse with 200 bays for pallet units in each of 14 product storage rows and 8 storage levels. In the simulation, 1,000 different products were used. The completion lists for the products were prepared according to the assumptions presented above. Figure 2 presents the results of the simulation of the order assembly time in a graphical form.

Figure 3 allows one to notice that methods the ABC and XYZ analyses, the method of free product storage places, the ABC analysis according to the criteria of popularity and number of sold items, and the ABC analysis together with the COI index according to the criteria of popularity and weight yield the results of the greatest range and standard deviation. This is also observable in the descriptive statistics presented in Table 1.

The variance analysis (ANOVA) was conducted in the analyses in order to test the significance of differences between the average values. The ANOVA analysis performs the distribution of data variances into two components: the component between groups and the component within the group. The index of Test F, which equals 148.23 in this case, is the ratio of the evaluation between the groups to the evaluation within the group. Since the value of the index p for Test F is lower than 0.05 then, it could be state with a 95% level of confidence that there is a statistically significant difference between the averages from the conducted analyses.

To state which groups statistically differ one from another, multiple comparisons were made using post hoc tests. To carry out multiple comparisons, Scheffe’s test, Tukey’s (HSD) test, Fisher’s (LSD) test, Bonferroni’s test, Newman-Keuls test and Duncan’s test were used. In the analysed case, Scheffe’s test was carried out – this test is considered to be one of the most conservative post hoc tests [25]. The result of the test is shown in Fig. 4.

![Graph](image-url)
Table 1. Descriptive statistics of the analyses performed for variant I [s]

<table>
<thead>
<tr>
<th>Description</th>
<th>Average</th>
<th>Standard Deviation</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABC combined with XYZ</td>
<td>1488.30</td>
<td>490.83</td>
<td>1479.60</td>
<td>470.20</td>
<td>2961.80</td>
<td>2491.60</td>
</tr>
<tr>
<td>Index COI</td>
<td>1092.06</td>
<td>367.62</td>
<td>1072.20</td>
<td>269.80</td>
<td>2422.80</td>
<td>2153.00</td>
</tr>
<tr>
<td>ABC by number of units sold</td>
<td>1348.97</td>
<td>432.03</td>
<td>1344.00</td>
<td>491.20</td>
<td>2646.20</td>
<td>2155.00</td>
</tr>
<tr>
<td>ABC by number of units sold and Index COI</td>
<td>1367.12</td>
<td>449.19</td>
<td>1357.40</td>
<td>499.00</td>
<td>3028.60</td>
<td>2529.60</td>
</tr>
<tr>
<td>ABC by capacity</td>
<td>1179.28</td>
<td>390.51</td>
<td>1141.60</td>
<td>205.20</td>
<td>2512.00</td>
<td>2306.80</td>
</tr>
<tr>
<td>ABC by capacity and Index COI</td>
<td>1180.41</td>
<td>412.38</td>
<td>1139.00</td>
<td>268.00</td>
<td>2401.20</td>
<td>2133.20</td>
</tr>
<tr>
<td>Method of free product storage</td>
<td>1532.15</td>
<td>510.02</td>
<td>1518.20</td>
<td>508.40</td>
<td>3141.80</td>
<td>2633.40</td>
</tr>
<tr>
<td>ABC by popularity</td>
<td>1387.24</td>
<td>453.13</td>
<td>1392.80</td>
<td>444.20</td>
<td>2610.60</td>
<td>2166.40</td>
</tr>
<tr>
<td>ABC by popularity and Index COI</td>
<td>1509.66</td>
<td>507.12</td>
<td>1508.00</td>
<td>473.40</td>
<td>3042.60</td>
<td>2569.20</td>
</tr>
<tr>
<td>ABC by weight</td>
<td>1162.95</td>
<td>367.17</td>
<td>1155.00</td>
<td>262.00</td>
<td>2399.20</td>
<td>2137.20</td>
</tr>
<tr>
<td>ABC by weight and Index COI</td>
<td>1162.29</td>
<td>394.50</td>
<td>1154.60</td>
<td>275.20</td>
<td>2409.80</td>
<td>2134.60</td>
</tr>
<tr>
<td>Total</td>
<td>1310.04</td>
<td>462.70</td>
<td>1271.80</td>
<td>205.20</td>
<td>3141.80</td>
<td>2936.60</td>
</tr>
</tbody>
</table>

Fig. 4. The result of the comparison of averages using Scheffe's interval with a 95% of confidence level
Comparison of the medians with the use of the Friedman test was the last part of the analysis. The null hypothesis assumed that all the attempts came from the population with the same median [26, 27]. The result of the test was 3727.91, where the p parameter equalled 0.0. This enabled the conclusion that the groups significantly differ one from another.

By comparing the average values and medians, it was stated that the best result were obtained when the products in the warehouse were distributed on the basis of the COI Index (average: 1092.0, median: 1072.2). Other methods yielded the following results: the ABC analysis according to the weight criterion produced an average of 1162.9 and a median of 1155.0; the ABC analysis according to the weight criterion combined with COI produced an average of 1162.3 and a median of 1154.6; the ABC analysis according to the volume criterion produced an average of 1179.3 and a median of 1141.6; the ABC analysis according to the volume criterion combined with COI produced an average of 1180.4 and a median of 1139.0. The methods show a statistical similarity and yield slightly worse results.

The ABC analysis combined with XYZ, the ABC analysis combined with the index COI according to the popularity criterion and the method of randomly assigned storage locations produced the worst results characterised by their high standard deviation and high average and median. Thus, for large warehouses, it is important to distribute products on the basis of the ABC analysis according to the criteria of weight, volume and the COI index. This method enables the obtaining of better results than in the case of randomly assigned storage locations by 28.72% on average.

4. Variant II

As in the case of Variant I, simulations were conducted for Variant II. As before, these were designed for a large warehouse with 200 spots for pallet units in each of 14 product storage rows and 8 storage levels. The assumption was that products had limited sensitivity to stacking; thus, during their completion it was necessary to change their order on the completion carrier to prevent their damage In this simulation, 1000 various products were used. The order lists for the products were prepared according to the established assumptions. Figure 5 presents the results of the simulation of the product completion time in a graphical form.

It is difficult to identify a significant difference between the sets of results of order completion time using various methods of product classification in the above figure. Descriptive statistics for the conducted analyses is presented in Table 2. The values which exceed the average are highlighted yellow.

For the performed analyses, the analysis of variance was performed in order to test the significance of differences between average values. The index of Test F, which equals 8.53 in this case, is the ratio of the evaluation between groups to the evaluation within the group. Since the level of probability for Test F is $p < 0.05$, we can state with a 95% level of confidence that there is a statistically significant difference between the averages from the conducted analyses.
Table 2. Descriptive statistics of the analyses performed for variant II [s]

<table>
<thead>
<tr>
<th>Method of product classification</th>
<th>Average</th>
<th>Standard deviation</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABC combined with XYZ</td>
<td>2041.17</td>
<td>1031.75</td>
<td>1858.27</td>
<td>517.60</td>
<td>8927.85</td>
<td>8410.25</td>
</tr>
<tr>
<td>Index COI word order consistency</td>
<td>1872.67</td>
<td>1101.33</td>
<td>1639.72</td>
<td>298.64</td>
<td>8443.14</td>
<td>8144.50</td>
</tr>
<tr>
<td>ABC by number of units sold</td>
<td>1960.62</td>
<td>1044.38</td>
<td>1804.89</td>
<td>491.20</td>
<td>8801.36</td>
<td>8310.16</td>
</tr>
<tr>
<td>ABC by number of units sold and index COI</td>
<td>2114.52</td>
<td>1181.52</td>
<td>1878.08</td>
<td>499.00</td>
<td>9649.18</td>
<td>9150.18</td>
</tr>
<tr>
<td>Method of free product storage places</td>
<td>2139.11</td>
<td>1033.57</td>
<td>1973.83</td>
<td>553.40</td>
<td>6698.40</td>
<td>6145.00</td>
</tr>
<tr>
<td>ABC by capacity</td>
<td>1872.95</td>
<td>1101.54</td>
<td>1605.07</td>
<td>205.20</td>
<td>9240.38</td>
<td>9035.18</td>
</tr>
<tr>
<td>ABC by capacity and index COI</td>
<td>1968.20</td>
<td>1180.27</td>
<td>1688.46</td>
<td>274.71</td>
<td>8263.58</td>
<td>7988.87</td>
</tr>
<tr>
<td>ABC by popularity</td>
<td>1964.75</td>
<td>1040.72</td>
<td>1759.65</td>
<td>477.80</td>
<td>8447.86</td>
<td>7970.06</td>
</tr>
<tr>
<td>ABC by popularity and index COI</td>
<td>2187.11</td>
<td>1141.98</td>
<td>1992.80</td>
<td>560.33</td>
<td>9171.19</td>
<td>8610.86</td>
</tr>
<tr>
<td>ABC by weight</td>
<td>1981.27</td>
<td>1156.62</td>
<td>1711.05</td>
<td>359.88</td>
<td>8941.33</td>
<td>8581.45</td>
</tr>
<tr>
<td>ABC by weight and index COI</td>
<td>1982.77</td>
<td>1189.4</td>
<td>1678.64</td>
<td>375.4</td>
<td>8592.83</td>
<td>8217.43</td>
</tr>
<tr>
<td>Total</td>
<td>2007.74</td>
<td>1114.84</td>
<td>1780.36</td>
<td>205.2</td>
<td>9649.18</td>
<td>9443.98</td>
</tr>
</tbody>
</table>

Fig. 5. Order completion time depending on the applied method of product classification, Variant II
Multiple comparisons were made with the use of the Scheffe post hoc test in order to identify which groups statistically differ one from another – the results of the test are shown in Fig. 6.

![Graph showing the results of the comparison of averages using the Scheffe's interval with a 95% confidence level.](image)

**Fig. 6.** The results of the comparison of averages using the Scheffe's interval with a 95% confidence level

On the basis of the conducted Scheffe test, it can be observed that as with Variant I, there is a similarity of results for the ABC analysis across all criteria and for the ABC analysis with the COI index according to weight and volume.

The last part of the analysis was the comparison of the medians with the use of the Friedman test. The null hypothesis assumed that all the attempts came from the population of the same median. The result of the test was 1121.65, in which the level of probability was $p = 0.0$. This enabled the conclusion that the groups significantly differ one from another. A graphical presentation of results in the form of a box-and-whisker plot is shown in Fig. 7.

By comparing the average values and medians, it can be stated that the best results were obtained when the products in the warehouse were located on the basis of:
- the COI index, average: 1872.67, median: 1639.72,
- the ABC analysis according to the number of items sold, the average: 1960.62, median: 1804.89.

The worst results were obtained using the ABC analysis combined with the COI index according to the popularity of products. The average value of the completion time was 2187.11 s, the median was 1992.80 s, and the result range was 8610.86. It can therefore be stated that for large warehouses and products with minimal suitability for stacking the best methods are those in which the volume and the weight of products are the decisive criteria, e.g. Index COI. This method allows better results than in the random assignment of locations method by 12.45% on average.
5. Conclusions

On the basis of the performed simulations, it has been stated that the analyses based on product properties such as the weight and volume yield higher effectiveness than the methods of random assignment of locations or classical analyses performed on the basis of the criteria of the number of items sold or product popularity. It has also been confirmed that the localisation of products in the warehouse whilst taking into consideration their volume and weight enables the improvement of the execution of numerous orders simultaneously.

If the suitability of the products for stacking is not a factor in the order execution process, then for large warehouses, it is important to distribute products on the basis of the ABC analysis according to the criteria of weight, volume and COI index. This method enables better results than the random assignment of locations method by 28.72% on average. However, for products with minimal stacking suitability, the best methods are those in which the decisive criteria are the volume and weight of products, e.g. the COI index. This method produces better results than the random assignment of locations method by 12.46% on average. The cost of the picking process is about 30–50% of all warehouse costs; therefore, decisions relating to the product layout planning each day plays the primary role with regard to operational decisions. The presented method can support that decision and decrease the risk of bad choices in the context of product location planning. On the basis of the results on this research, it can be anticipated that using this software could reduce total warehouse costs by around 10-16%. The research results are, therefore, of significant importance for warehouse management. At present, implementation works are being carried out within the kinds of companies that would benefit from such improvements. The results of the presented research may constitute a reliable basis for aiding the selection of product classification methods when planning the locations of different products within a warehouse.
References


