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**ACTIVE POWER MEASUREMENT BASED ON DIGITAL PROCESSING OF  
VOLTAGE AND CURRENT SIGNALS**

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**WYKORZYSTANIE CYFROWEGO PRZETWARZANIA SYGNAŁÓW DO  
POMIARU MOCY CZYNNEJ****Abstract**

The concept of active power measurement based on real-time digital processing of voltage and current signals is best introduced with a digital approximation definition. In this paper, the authors propose an integral calculation method based on Gregory's expression, correction of integration period estimation and integration result correction taking values related to the ends of the integration period into account. In the described method, the integral is calculated between two consecutive maximums of the voltage and current product. Digital dependencies that are easy to implement are used for the evaluation of the active power value. To measure the voltage and current signals, the authors used a system with a digital signal processor and an A/D converter with a multiplexer followed by dedicated digital filters. The active power measurement method was checked by building an instrument simulation model.

**Keywords:** active power, digital filtering FIR

**Streszczenie**

Pomiar mocy czynnej za pomocą systemu pomiarowego realizującego przetwarzanie sygnałów napięcia i prądu w sposób cyfrowy wymaga przybliżenia zależności definicyjnych. W artykule zaproponowano wykorzystanie wzoru Gregoriego dla celów obliczenia całki w zależności definicyjnej mocy czynnej. Dodatkowo proponowana jest poprawka związana z wyznaczeniem przedziału całkowania i dotyczy wyznaczania maksimumów sygnałów. Drugim proponowanym rozwiązaniem jest użycie jednego przetwornika A/C z multiplexerem, a następnie użycie filtrów cyfrowych w celu wyrównania opóźnień pomiędzy próbkowanymi sygnałami. Proponowane rozwiązania umożliwiają zbudowanie prostego systemu pomiarowego pracującego w czasie rzeczywistym.

**Słowa kluczowe:** moc czynna, filtr cyfrowy FIR

## 1. Active power evaluation from samples of voltage and current signals

Active power is one of the most frequently measured electrical values. The value of this power is especially helpful in the measurement of the voltage and current values, mainly in the field of simplifying their calculations. The definition of the active power for the finite-duration of signals relating to voltage and current is as follows:

$$P = \frac{1}{t_b - t_a} \int_{t_a}^{t_b} u(t)i(t)dt \quad (1)$$

where:  $(t_a, t_b)$  – time interval of voltage  $u(t)$  and current  $i(t)$  observation

Digital implementations of the Eq.(1) is based on the adding up of samples of voltage  $u_k$  and current  $i_k$  signals, [3]. This means calculating the integral Eq.(1) as adding products of voltage and current samples and can be realised by means of any method of numerical integration e.g. by rectangles method. Simple algorithm is easy to implementation but the integration limits with resolution referred to the sampling period leads to relatively large calculation errors. These errors are limited by reducing the sampling period which leads to a demand for greater computing power provided in acquisition card and fast A/D converters. It is also possible to determine the integral Eq.(1) applying more complex algorithms e.g. the trapezoidal algorithm [1, 2]. However, it is worth to nothing here that method based on many nodes is leading to an equally simple algorithm as the method of rectangles [4, 7]. Based on [4, 5, 7] an even more accurate method of setting the integral from the relation Eq.(1) was suggested using Gregory's interpolation formula:

$$K = \Delta t \cdot \left[ \frac{9}{24}u_0i_0 + \frac{28}{24}u_1i_1 + \frac{23}{24}u_2i_2 + u_3i_3 + \dots + u_{N-3}i_{N-3} + \frac{23}{24}u_{N-2}i_{N-2} + \frac{28}{24}u_{N-1}i_{N-1} + \frac{9}{24}u_Ni_N \right] \quad (2)$$

where:  $\Delta t$  – sampling period of voltage and current signals.

The method significantly improves the accuracy of the calculation of the integral, for example [5, 7], but for high accuracy of the calculation of active power Eq. (1), it is also necessary to determine the range  $(t_a, t_b)$  more precisely. For this purpose, the choice of integration ranges between consecutive maximums of signals  $u(t)$  and  $i(t)$  and the correction considering the field between estimates of maximum and sampling places are proposed. This correction is presented in Fig. 1.

Places in which the maxima appeared were fixed by appointing their estimates  $t_{\max}^{\sim}$  as the values of the maxima of parabolas passing through the three nearest registered points Eq. (3):

$$t_{\max}^{\sim} = \frac{\Delta t}{2} \cdot \frac{p(n_{\max} - 1) - p(n_{\max} + 1)}{p(n_{\max} + 1) - 2p(n_{\max}) + p(n_{\max} - 1)} \quad (3)$$

where:  $n_{\max}$  – place where the maximum appears  $p(n_{\max}) = u_{\max} \cdot i_{\max}$ .

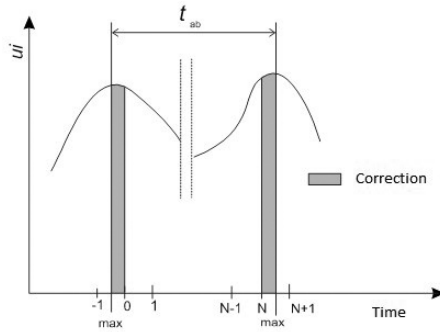


Fig. 1. Method of active power measurement (1) with correction

Correction including the displacement of the maximum with regard to sampling moments is received from the following relation (4):

$$k = \frac{p(n_{\max})}{2} \cdot \frac{p(n_{\max} - 1) - p(n_{\max} + 1)}{p(n_{\max} + 1) - 2p(n_{\max}) + p(n_{\max} - 1)} \quad (4)$$

Using Eqs. (2), (3), (4) we receive the following active power formula:

$$P = \frac{1}{t_{\max 2}^{\sim} - t_{\max 1}^{\sim}} (k_1 + k_2 + K) \quad (5)$$

The algorithm that determines the active power from formula (5) is as follows:

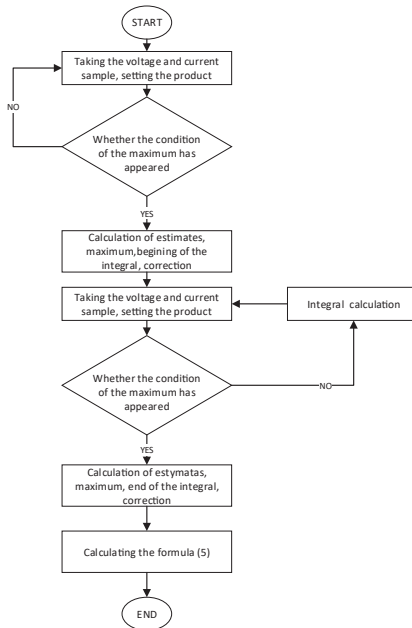


Fig. 2. Active power measurement algorithm according to Eq. (5)

The loop is used to set the integral for the sample indicator from 3 to N-3; however, the beginning and the end of formula Eq. (3) is determined in the part in which correction, associated with the estimation of the maxima, is calculated.

## 2. Execution of active power measurement

Measurement of the active power according to Eq. (5) requires obtaining the sequence of the voltage and the current samples that are taken at the same time. This is possible with some measuring instruments which simultaneously samples two signals. A/D converter and a multiplexer present construction of such an equipment.

Unfortunately, there is a problem referring to the time delay between samples of voltage and current. In order to avoid this problem the measurement of voltage and current signals in identical time intervals  $\Delta t$  was proposed.

It is worth noting here that measurement is simple for hardware implementation and, in addition, it does not impose a large dynamic requirements on the multiplexer. The synchronisation of samples is achieved by using digital filters with finite and symmetrical impulse responses [6]. A flowchart of the active power measurement with the used filters is presented in Fig. 3.

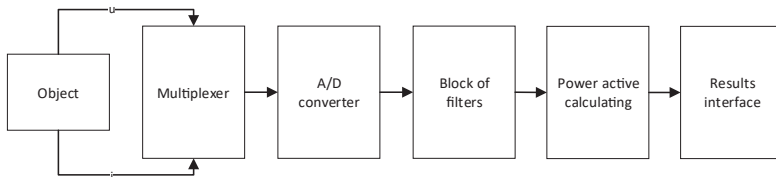


Fig. 3. Active power measurement with a single A/D converter with a multiplexer

## 3. Model testing

Prior to the construction of the device, the presented algorithm for active power measurement was tested by performing a sequence of experiments on the simulation model. The Matlab environment was used for testing of the model. After checking the correctness of the model, the device was able to measure the active power. In the second step, research was conducted in order to determine both the measurement accuracy and the influence on the accuracy of structural parameters.

Researches were carrying out for one sampling rate of 1 kHz signals. Depending on the chosen method of power calculation 1 kHz signal requires 2 kHz A/D converter which is critical condition of the program as the measurement should be made in time smaller than 500  $\mu$ s. A sinusoidal signal was used as a test signal. The defined error during testing was calculated from the following relation:

$$\delta = \left| \frac{P_d - P_w}{P_z} \right| \quad (6)$$

where:

$P_d$  – active power determined on the basis of the definition (1),

$P_w$  – active power determined during simulation,

$P_z$  – active power value corresponding to the range of tested model.

It can be observed that the measurement error decreases as the number of bits of the converter increases; however, this decline becomes insignificant as the number of bits gets to twelve or more. A small value of error of less than 0.5%, even for small number of bits, is the result of averaging properties of integration arising from definition Eq. (1), see Fig. 4.

This curve suggests the possibility of limiting the number of bits in the A/D converter to obtain a sufficiently small expected error. Fig.5. shows the relationship between the measurement error and the frequency of voltage and current signals for an A/D converter with the resolution of 12 bits. The big change of the error above 100 Hz indicates that the integrating algorithm already introduces a significant error. Above 250 Hz, the algorithm may not perform correctly. This is due to an insufficient number of samples being processed at the range for which the measurement is implemented.

Examining the error size which is dependent upon the angle of the phase shift between the voltage and current signals did not show noticeable changes in the value of the error from the angle of phase shift.

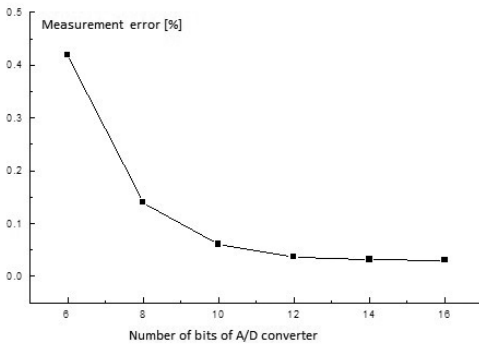


Fig. 4. Relationship between the measurement error and the number of bits of A/D converter

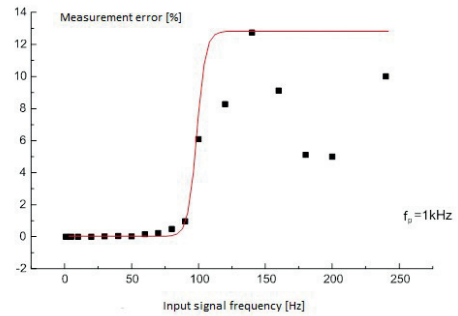


Fig. 5. Relationship between the measurement error and the frequency of voltage and current signals

#### 4. Summary

The presented results of the model testing confirm the accuracy of the solution of the active power measurement by using an A/D converter with a multiplexer. These formed the basis for the construction of the instrument based on a real-time digital signal processor.

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