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DIFFERENT ASPECTS OF SUPPORTING GROUP
CONSENSUS REACHING PROCESS UNDER FUZZINESSRÓŻNORODNE ASPEKTY WSPIERANIA PROCESU
OSIĄGANIA KONSENSUSU W GRUPIE
W WARUNKACH ROZMYTOŚCI

Abstract

In this paper we present human-consistent approach of multi-model consensus reaching process supporting by group decision support systems. We consider the idea developed by Kacprzyk and Zadrozny [9, 10, 12] which is related to the “soft” consensus, and where the core of the system is based on fuzzy logic. Essentially, we attempt to stress the multi-model architecture of considering system and distinguish several aspects, i.e. model of agent, model of moderator, model of consensus achievement. Moreover, we present a novel concept based on fair consensus as a meaningful point of further development.

Keywords: soft consensus, decision support systems, consensus reaching process, fairness

Streszczenie

W niniejszym artykule zaprezentowano wybrane aspekty złożonego procesu osiągnięcia konsensusu nadzorowanego przez systemy wspierające podejmowanie decyzji w grupie. Rozważono ideę opracowaną przez Kacprzyka i Zadroznygo [9, 10, 12] powiązaną z terminem „miękkiego” konsensusu, opartą na logice rozmytej. Przedstawiono złożoną architekturę rozważanego systemu oraz wyróżniono w nim kilka aspektów, tj. model agenta, model moderatora, model osiągnięcia konsensusu. Ponadto, zaprezentowano nową koncepcję opartą na sprawiedliwym konsensusie jako punkt wyjścia dla dalszych badań.

Słowa kluczowe: miękki konsensus, systemy wspierające podejmowania decyzji, proces osiągnięcia konsensusu, sprawiedliwość

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1. Introduction

Currently, almost everything that a human being does involves making some decisions. In fact, decision making is as old as conscious is human existence what boils down to the statement that it is an ubiquitous process which will never lose in meaning. Even though the best solution can be found by an entity or a group of individuals, the essence is always the same: there are some options to choose between and only one has to be chosen. Obviously, decision makers do not choose in a random way. Taking into account the fact that they always weigh costs and benefits, their decisions have to fulfill certain conditions of rationality. Hence, the theory about decision making calls this universal process as the *goal-directed behaviour in the presence of alternatives* [2].

We accepted the statement that the group of individuals (experts, agents) is known to be an effective organ in decision making process. Actually, it allows to make either more comprehensive analysis of the problem, as a result of larger amount of experiences and wider view of different aspects or a thorough examination considering more details rather than observed by an entity. Therefore, the *group decision making process* will be the groundwork of our further consideration.

At the first stage of the process individuals provide their preferences as to the particular pairs of options (this setting has its origin in *social choice theory*, hence the pairwise comparison). Later, all particular opinions are taken into account and aggregated to the one, common group decision. What matters here is that the main goal of the group decision making process is to achieve consensus in the sense of the agreement of group members as to the final decision. Thus, the decision problem discussed comprises of two levels [3]: firstly – making individual decision of each participant, secondly – developing of joint solution in the spirit of *consensus reaching process*. This distribution of the process describes its multistage and dynamic character defined as an interactive and iterative process over the time span. Obviously, model of the consensus reaching process makes sense only if individuals are able to negotiate and change their testimonies [16]. In each stage of the process the level of consensus can be measured. It is understood as a measure of distance between individuals and gives the agreement a topological meaning.

Moreover, there are many tools and methods which support and simplify consensus reaching process. They concern to defining decision problem as well as to data analysis, knowledge acquisition, leading the discussion or elaborating the agreement. In spite of assignment to different functions, they all have collaborative name – *group decision support systems* (GDSS). An overall structure of GDSS is exemplified in Section 2.

This paper shows several author's approaches of these systems which support group consensus reaching process. What matters here is that we do not directly compare them in the sense of their efficiency exemplified on specific data but make various reasonable assumptions. We attempt to introduce the reader basics of our novel methodology which is constantly under our consideration and will be expanded to numerical results in the following articles. An overall review of consensus reaching process could be omitted, as our investigation is strictly based on one idea developed by Kacprzyk and Zadrozny [9, 10, 12] which is related to the "soft" consensus, and where the core of the system is based on fuzzy logic (Section 3 and 4).

2. An overall structure of the system

The name of this system is very descriptive. “A GDSS is a hybrid system that uses an elaborate communications infrastructure and heuristic and quantitative models to support decision making” [18]. According to Sprague, DSS “is comprised of three set of capabilities: database management software, model base management software, and the software for managing the interface between the user and the system, which might be called the dialogue generation and management software. These three major subsystems provide a convenient scheme for identifying the technical capability which a DSS must have.” [17 – p. 14].

The overall structure of the system is shown in Fig. 1. Its core is composed of preference testimonies and consensus measurement modules, but the discussion and external information sources are also treated as relevant part of the system.

We will describe very briefly the subsequent components shown in Fig. 1. *Setting the agenda* – this is the first stage of the consensus reaching process which concerns definition of the decision making problem. The representation of structure of alternatives (e.g. hierarchical structure which is discussed in section 4) is denoted as *domain ontology* (*dom \square ont*) while the *consensus ontology* (*cons \square ont*) defines main concepts of the consensus reaching process. The *discussion* is meant as a way to clarify the preferences of the decision makers as to the every pair of alternatives, exchange of the knowledge and advocate different opinions. “NLG” is here an example of the natural language generation task. This system facilitates the generation of arguments in the form of natural language expressions called “comparatives” (ordering between two options regarding to the degree to which they are closer to their preferences). After discussion the individuals express their preferences in the form of *preference relation*. The explicit explanation of this part, as well as measurement of consensus degree is mentioned in section 3 of this article. If a satisfactory consensus has been obtained the session ends, otherwise another round of discussion is set up and some clues are made by the system in order to help guide the process more efficiently.

To clarify, initially preferences of individuals are very far from each other and this system aims at minimize this distances and lead the group closer to the consensus in the most effective and efficient way. The integral part of this computer-based system is a *moderator* which, in principle, plays a main role with regard to his indirect influence on the quality of final decision. First of all, moderator checks whether consensus is reached (and process can be stopped) and ensures efficient course of process (in the direction of increasing consensus), especially by supporting the discussion in the group which is known to be a central part of the consensus reaching process. Moderator, by measuring the distance between individuals, monitors the relation in the group. A very important thing is that a moderator makes arguments and convinces proper decision makers to change their testimonies rather than puts a pressure to accept e.g. some pointing will. This proceeding affects on the sense of satisfaction among the group members which, according to the psychological research, has a direct influence on higher *decision quality*.

By the *feedback information generation* we understand the fact that the system confronts the individual preferences relations and the list of options submitted by the decision makers during the discussion. Thanks to that some key components or attributes of the alternatives may be identified, which play an important role in the disagreement, e.g. which individuals are the most troublesome (stubborn) or which alternatives are the most crucial.

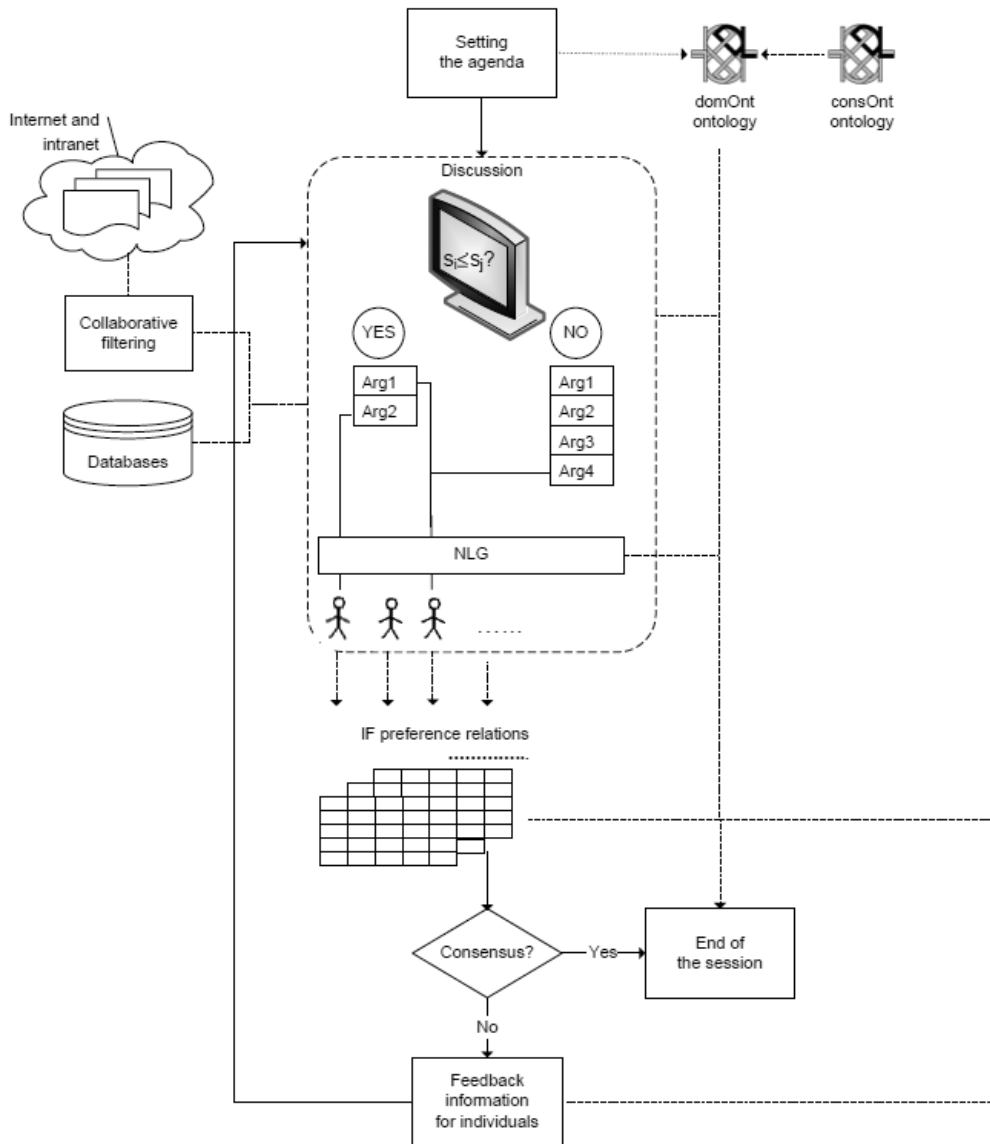


Fig. 1. The structure of proposed system [7]

Rys. 1. Struktura proponowanego systemu [7]

Moreover, *external information sources* and *collaborative filtering* compose an important part of our consensus reaching model. By means of the former we assume that arguments in the discussion may be supported by all textual documents available on the Internet. The latter is used in order to make the flow of information more efficient (some documents which have been found relevant by other individuals of similar profiles are suggested to the individuals

looking for documents) [7]. It seems that such a combination of tools and modern knowledge will help develop an innovative human-consistent system for supporting consensus reaching process.

3. Fuzzy logic as the core of the multi-model system

We discuss a consensus reaching process in a group of experts. To simplify, we attempt to make preferences of the individuals more similar and, in fact, get the experts closer to the consensus in the sense of agreement. Therefore, we consider a multistage hierarchical model, with the superior aim meant as the highest level of consensus reached by the group and certain inferior goals depending on the concerning approach i.e. the way of aggregation of individual preferences to the group decision. What is the most important here is that the core of this system is the preferences modeling and consensus assessment module which are based on fuzzy logic.

We present the brief description from the comprehensive approach introduced by Kacprzyk and Zadrozny [10]. Basically, the following structure has been assumed. There is a finite set of $N \geq 2$ alternatives, $S = \{s_1, s_2, \dots, s_N\}$, and a finite set of $M \geq 2$ individuals $E = \{e_1, e_2, \dots, e_M\}$. Each individual $e_M \in E$ expresses his/her preferences as to the particular pairs of options in the form of individual *fuzzy preference relation* R_m in $S \times S$, and its membership function $\mu_{R_m} : S \times S \rightarrow [0, 1]$. Namely, $\mu_{R_m}(s_i, s_j) > 0.5$ indicates the preference degree of an alternative s_i over an alternative s_j , and $\mu_{R_m}(s_i, s_j) < 0.5$ indicates, properly, the preference degree of an alternative s_j over an alternative s_i . The third possible relation represented by $\mu_{R_m}(s_i, s_j) = 0.5$ is also acceptable and denotes the indifference between two considering alternatives s_i and s_j . Usually, R_m is assumed reciprocal, i.e.:

$$\mu_{R_m}(s_i, s_j) + \mu_{R_m}(s_j, s_i) = 1 \quad (1)$$

holds.

Normally, these testimonies are entirely different in the beginning, but during subsequent steps of the consensus reaching process they are being changed, by some argumentation, prepositions, mutual concessions, etc. [12]. What can be problematic here is how to create the matrix of accordance which defines the evaluation of consistency or non-consistency of any individual as to the some “typical” expert. Basically, we need to determine a similarity of every decision maker as to the some abstract entity or one of the existent individual (i.e. some average decision maker).

As we mentioned before, consensus reaching process derives from social choice theory which initially assumed only two ways of consensus measure: 1 which denoted total agreement as to the final decision and 0 which meant that the consensus was not reached by the group. Unfortunately, this scenario does not overlap the real life, because the human perception of the consensus is definitely much “softer”.

The *soft consensus*, which represents more realistic attitude, can lead to solve in a more effective way the group decision making tasks by using fuzzy logic models. For that reason, we assume a conceptual human-consistent framework proposed by Kacprzyk

and Fedrizzi [6, 7], and Zadrozny [1]. According to their research, consensus is meant as a certain *degree of agreement*. It means that, except none or total agreement between agents as to the chosen solution, this approach allows to some partial, acceptable consistency in the range $[0, 1]$ [11].

The proposed idea is meant basically as an agreement of a considerable majority of individuals with regards to a considerable majority of alternatives. This operational definition of consensus can be, for instance, expressed by a linguistically quantified preposition: “most of the individuals agree in their preferences to almost all of the options”, and the consensus degree (from $[0, 1]$) is computed as the truth value of this statement. Basically, the calculation of truth (validity) can be done by using Zadeh’s classic calculus of linguistically quantified prepositions [21] or Yager’s OWA (ordered weighted average) operators [20]. Notice, that to define a fuzzy majority for measuring a degree of consensus the application of *fuzzy linguistic quantifiers* (most, almost all etc.) has been performed. The computations of this relative type of linguistic quantity can be also handled via Zadeh’s classic calculus of linguistically quantified prepositions. Regardless of the way of implementation, the main condition of this novel approach is that it definitely overcomes the conventional concept in which full consensus occurs only when “all the individuals agree as to the all the alternatives”, what is unrealistic in practice [12].

Basically, this novel idea has been successfully implemented by Kacprzyk and Zadrozny [11] in a decision support system for the group consensus reaching process. First of all, the consensus degree plays itself an important role in guiding the consensus reaching process, because it exemplifies some satisfactory agreement among the group of experts as to the final decision. Moreover, this approach derives additionally some partial indicators of consensus, like i.e. the personal consensus degree or the option consensus degree. These consensus indicators point out the most controversial alternatives and/or individuals isolated in their opinions. Thus, they are used to facilitate the work of the moderator and make it more effective by providing him some hints as to the most promising directions of a further discussion.

Finally, this linguistically quantified prepositions was extended and successfully implemented by Kacprzyk and Zadrozny [9] to the prototypical forms of *linguistic data summaries* (introduced by Yager [19] and considerably advanced by Kacprzyk, Yager and Zadrozny [8]) which indicate relations between individuals and options in a natural language. On the basis of large data sets they advocate the use of linguistic data summaries as linguistically quantified prepositions in consensus reaching process which may help the moderator get a deeper understanding of correspondence among individuals and their testimonies.

4. Efficient degree of soft consensus

The previous approach is based on the democratic group of individuals without distinguishing their role during decision making process. Likewise, the set of alternatives do not take into account the type of possible options concerning by the group, which has a large impact on the quality of final decision. Namely, the more relevant options are taken into consideration, the higher is the quality of final decision, otherwise, the more irrelevant

options are considered by the group, the greater is the confusion of information and so the more difficult is to find a final solution of the problem. The similar situation concerns the set of individuals while their importance is taken into consideration.

The consensus reaching process, carried out by a moderator, is expected to get the preferences of the group of experts as close to each other as possible in the most efficient way. Therefore, Kacprzyk and Zadrożny [10] proposed a model based on importance of individuals and relevance of alternatives. The new degree of soft consensus is meant here as a degree to which: “most of the important individuals agree in their preferences to almost all of the relevant options”.

The relevance of alternatives is assumed to be given as a fuzzy set B determined as a set of alternatives S such that $\mu_B(s_i) \in [0, 1]$ is a degree of relevance of alternative s_i , from 0 for fully irrelevant to 1 for fully relevant, through all intermediate values. The relevance b_{ij} of a pair of alternatives $(s_i, s_j) \in S \times S$, may be defined, as:

$$b_{ij}^B = \frac{1}{2} [\mu_B(s_i) + \mu_B(s_j)] \quad (2)$$

for each i, j , where $i \neq j$. Evidently $b_{ij}^B = b_{ji}^B$, for each i, j .

Analogously, the importance of individuals, I , is defined as a fuzzy set in the set of individuals E such that $\mu_I(e_m) \in [0, 1]$ is a degree of importance of individual e_m , from 0 for fully unimportant to 1 for fully important, through all intermediate values. Thus, the importance b_{mn}^I of a pair of experts, $(e_m, e_n) \in E \times E$, may be defined as:

$$b_{mn}^I = \frac{1}{2} [\mu_I(e_m) + \mu_I(e_n)] \quad (3)$$

After implementation of those additional assumptions, the degree of consensus is derived on several levels. We describe it briefly, only to ensure the overall view on this procedure. First, for each pair of experts (e_m, e_n) and each pair of options (s_i, s_j) a degree of agreement $v_{ij}(m, n)$ is obtained as:

$$v_{ij}(m, n) = \text{Aggrm}(\mu_{R_m}(s_i, s_j), \mu_{R_n}(s_i, s_j)) = \begin{cases} 1 & \text{if } \mu_{R_m}(s_i, s_j) = \mu_{R_n}(s_i, s_j) \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

Secondly, for each pair of individuals (e_m, e_n) a degree of agreement $v_{Q_1}^B(m, n)$ as to their preferences between Q_1 (linguistic quantifier, i.e. “most”, “almost all”) pairs of relevant alternatives is derived as:

$$v_{Q_1}^B(m, n) = \mu_{Q_1} \frac{\sum_{i=1}^{N-1} \sum_{j=i+1}^N [v_{ij}(m, n) \wedge b_{ij}^B]}{\sum_{i=1}^{N-1} \sum_{j=i+1}^N b_{ij}^B} \quad (5)$$

Thirdly, these degrees are aggregated to obtain a degree of agreement $\text{con}(Q_1, Q_2, I, B)$ of Q_2 (linguistic quantifier, i.e. “most”) pairs of important individuals as to their preferences between Q_1 (linguistic quantifier) pairs of relevant alternatives. This is meant to be the sought *degree of consensus*:

$$\text{con}(Q_1, Q_2, I, B) = \frac{2}{M(M-1)} \frac{\sum_{m=1}^{M-1} \sum_{n=m+1}^M [v_{Q_1}^B(m, n) \wedge b'_{m,n}]}{\sum_{m=1}^{M-1} \sum_{n=m+1}^M b'_{m,n}} \quad (6)$$

We assume, that the use of hierarchical structure of experts and alternatives makes the consensus reaching process more efficient. We also suppose that pursuit to the agreement among the more important individuals and more relevant options allows to obtain higher degree of consensus rather than degree of soft consensus gained within this distinction.

5. Fair consensus degree

In order to elaborate the most human-consistent approach, the double nature of the decision problem has to be included in our group decision making model. Hence, *task orientation group* and *interpersonal orientation group* are distinguished [4]. If we are talking about the first one, the goal of the decision making process is the selection of the best option, so it is not so important if the individuals differ in their preferences as to the possible alternatives. However, we constantly consider the group with interpersonal orientation, where the solution of the decision making problem is only a minor goal. Here, the priority is to ensure a good relation within the group members during decision making process and to achieve consensus in the sense of some satisfactory agreement.

With regard to the necessity of *group specific knowledge*, we noticed, that two previous approaches (Section 3 and 4) do not guarantee equal participations of all decision members during the consensus reaching process. Indeed, in the beginning of the session preferences of every individual are taken into account, but at the later stages when the moderator gets them closer to the consensus by argumentation and persuasion as to the most promising directions, individuals which are isolated in their opinion are omitted. Unfortunately, this outsiders do not sense the satisfaction of the discussion what affects on the effectiveness of entire group. Of course, it does not exclude the consensus achievement, but decreases the opportunity of many, further activities, i.e. practical implementation of the final decision, survival of the group in the long time period, etc. [15]. Therefore, all of these socio-psychological aspects forced us to seek for a novel approach of consensus degree which will consider the satisfaction of every individual throughout the consensus reaching process.

Referring to social sciences, individuals make a *rational decision* [5], which means that they always weigh costs and benefits during decision making process. Moreover, the similar conclusions come out of *reciprocity rule* which briefly imposes that every co-operative action should be reciprocal (given back). To adapt this real group behaviour,

the new model is based on *bicriteria fair optimization*. As in the previous approaches, the central role of the group decision support system plays a discussion, run by moderator. But this time, the moderator not only argue and persuade proper expert to change their testimonies in order to looking for the best possible decision, but also has to ensure fairness and equity (justness) of the process. Moderator can not omit the experts who are isolated in their opinions as to the rest of the group members, quite the contrary it has to convince them to change their previous preferences. This attitude undoubtedly carries out one of our assumption, namely, active participation of every individual during the entire consensus reaching process.

The main core of this concept is the disposition to concessions defined for every pair of individuals. It may be considered as to the *fair resource allocation* problem extended by Ogryczak [13, 14]. Let us assume that the entire group has a resource equals 1 and a moderator, during consensus reaching process, tries to allocate it similarly on all of the participants. Hence, in every stage of this dynamic agreement reaching process, moderator can define a problem (difference between current resource allocation and fair resource allocation) and persuade proper individuals to change their preferences (i.e. increase disposition to concessions) and, finally, get the group closer to fair consensus. Regardless the choice of socio-psychological concept required in this methodology, the main goal is to achieve such a degree of consensus that the reached decision will be *highly justified*. We assume, that the novel degree of *fair consensus* will be much higher than a degree of *efficient consensus* presented in the previous chapter of this article. Essentially, this is the main basis of our current research and will be precisely defined in the following article.

6. Concluding remarks

The purpose of the paper was only to present selected approaches of supporting group consensus reaching process based on fuzzy logic. We considered either soft consensus models proposed and successfully developed by Kacprzyk and Zadrozny or quite new, conceptual approach based on fair and equitable degree of consensus. All of them are novel and each following described in this paper supposes to take into account more socio-psychological aspects of group behaviour, and de facto alleges to be more human-consistent what is very desirable in any “intelligent” system.

To stress the meaning of developed advances, we assume that each following approach characterizes the higher degree of consensus rather than the previous one. Namely, degree of classical soft consensus is lower than the degree of efficient consensus which included, additionally, hierarchical structure of experts and alternatives. Finally, the degree of efficient consensus is lower than the degree of fair consensus derived by fair and equitable resource allocation.

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