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GRAMMATICALIZATION PATHS AND CHAOS: DETERMINISM AND UNPREDICTABILITY OF THE SEMANTIC DEVELOPMENT OF VERBAL CONSTRUCTIONS (PART 1 – CHAOS IN MATHEMATICS)

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Abstract

This paper demonstrates that by applying Chaos Theory to the modeling of the evolution of verbal forms and verbal systems, it is possible to view classical grammaticalization paths as universal, and conceal this deterministic assumption with the unpredictability of concrete grammatical developments. The author argues that such an explanation is possible because traditional grammaticalization paths do not represent realistic cases of grammatical evolutions, but rather correspond to abstract and non-realistic deterministic laws which codify the order of the incorporation of new meanings to the semantic potential of a gram. Therefore, from a synchronic perspective, they can be used to represent the semantic potential of a form as a map or a state. In contrast, a realistic development emerges as a trajectory connecting such maps or states. Consequently, the cross-linguistic typological model of realistic evolutionary processes of a certain type corresponds to a state-space – it is a cluster of all possible trajectories the grams of a certain class can travel. In this article – the first of the series of three papers – the main tenants of Chaos Theory will be discussed.

1. Introduction

Various empirical studies have shown that, in languages of the world, components of verbal systems evolve by following certain principles. Given their common graphical representation as unidirectional trajectories, scholars have referred to these principles

as “paths.” In general terms, a path provides a model of the semantic growth of grammatical forms belonging to a certain taxonomical class. It depicts an ordered evolution of verbal constructions of a determined type from lexical, semantically transparent and possibly iconic periphrases to core, untransparent, grammatical categories, such as aspect, taxis, tense or mood (Heine, Claudi, Hünemeyer 1991ab; Bybee, Perkins, Pagliuca 1994; Dahl 2000b; Hopper, Traugott 2003; Heine, Kuteva 2006, 2007; Narrog, Heine 2011).¹

Paths have been induced from extensive empirical studies in which numerous languages of divergent families have been analyzed. Given the impressive amount of data supporting certain clines, paths have been viewed not only as typological strong tendencies but also as *quasi* universal (Bybee, Perkins, Pagliuca 1994: 14–15; Hopper, Traugott 2003: 99–100). This *quasi* universality is implied by two substantial characteristics of the theory of verbal grammatical paths (Path Theory),² as posited by Bybee, Perkins, Pagliuca (1994: 11–14), namely by their source determination and unidirectionality. According to the former, the source meaning determines the grammaticalization path of a gram. According to the latter, a grammaticalization path is a cline of consecutive stages and is “travelled” exclusively in one direction. Both properties predict that evolutions of typologically similar inputs are also similar topologically – that is, they are expected to follow an analogous pathway. Put differently, the theory postulates a great cross-linguistic similarity (or convergence) in the trajectories of similar sources. However, although Bybee, Perkins and Pagliuca (1994) use the term “universal path” when referring to posited representations of verbal developments, they typically mean ‘greatly or commonly similar paths’. Most importantly, the “universality” does not prohibit divergences from canonic scenarios, motivated by language-specific idiosyncrasy, but uniquely proposes a great resemblance in grammatical evolution of verbal forms whereby certain trajectories are “well-travelled” (Bybee, Perkins, Pagliuca 1994: 14–15, 23, 27, 104).

If Path Theory is understood in this “classical” manner, two problems arise. On the one hand, if evolutions are quasi universal in the sense that their universality and, hence, determinism are only statistically true (which accounts for a large majority of cases, albeit not for all of them; cf. Newmeyer 1998: 275; Traugott 2001: 3), one may question – or, at least, have some reservations about – the epistemological value of the posited clines. As already explained, paths are typically comprehended as inductive generalizations built on the available empirical evidence. In this manner, they constitute hypotheses about robust tendencies (Bybee, Perkins, Pagliuca 1994: 104–105; Traugott 2001: 1). If they are viewed as mere propensities devoid of any rule-like status, their explanatory power is weak. Since in the construction of paths

¹ Obviously, the development of grammatical formations does not cease here, but continues until a construction is either lost or recycled in new locutions.

² The summation of such evolutionary scenarios that schematize a semantic and functional development of verbal constructions will be referred to as Path Theory. Of course, the term itself is an artificial *ad hoc* invention and does not exist as such. It is used in this article to encompass a group of the most prominent linguists that work in the area of verbal evolutionary typology or the semantic and functional development of verbal grams.

an immense fragment of reality is ignored, it can be argued that the statistics on which such paths have been built uniquely reflect a minimal portion of real-world data and, hence, their relevance is almost insignificant. To be exact, as paths have been derived from evidence available in approximately one hundred languages, the vast majority of linguistic systems have been ignored. Moreover, and even more importantly, there is no certainty that, future and past data – either currently lost or still unavailable – will confirm the proposed clines. Consequently, since a great (if not infinite) portion of the evidence is ignored, nothing prohibits the statistical universality of paths to drastically change in light of new data. Without an inductive move whereby limited evidence is upgraded to the status of a law, the significance of any generalization is rather minimal.

On the other hand, if someone understands the paths as *absolutely* universal – and hence deterministic – he or she likewise faces several problems. In general terms, three major objections may be formulated against a strict deterministic view of paths. First, various irregular evolutionary cases have been reported. For instance, scholars have already noticed that other elements of the system may importantly modify the strictly linear orientation of universal trajectories, leading to outcomes that, in extreme cases, are not expected by the path model (cf. Bybee, Perkins, Pagliuca 1994: 14–15, 90–91; Dahl 2000a: 10–11). Second, if the development of a gram were universal and the laws were deterministic, one would expect that each single modification of the system would equally be absolutely determined and, thus, the system itself, in its integrity, would be subject to a deterministic evolution. However, in contrast to this alleged deterministic nature of paths, the predictability of concrete grammatical evolutions seems to be much weaker. In fact, the exact behaviour of a linguistic cannot be estimated with an absolute or even relative certainty. Lastly, third, if the entire evolution were to be strictly deterministic and linear (thus leading to infallible predictions) constructions deriving from the same original input (i.e. from an initial periphrasis that emerged in the mother language) should not acquire different properties at later stages of their independent evolutions in daughter languages. If the input locution follows a deterministic evolution, the development should be not only similar but also identical everywhere, i.e. in all dialects and languages, once the original vernaculars have been emancipated as independent linguistic systems.

Consequently, by adopting either a *quasi* or *absolutely* universal position, linguists face two different problems. On the one hand, if clines are taken as *quasi* (i.e. statistically) true, their epistemological value is weak: they are merely (accidental) generalizations without any rule-like force.³ They can possibly be revoked if, for instance, future evidence contradicts them for any reason. There is nothing *per se* that prohibits future data from pointing to different tendencies. On the other hand, if paths are understood deterministically as *absolutely* universal, various

³ This does not have to be an exaggeration. Since such laws are understood as purely inductive, they are sensitive to the criticism in the spirit of Popper. Since they are statistical, if the sample of items under study changes, the statistic outcome may likewise be affected.

contradictions appear: grams sometimes do not follow the expected trajectory and the predictability of grammatical evolution (both as far as the past and future states of a language are concerned) is impossible. As scientists, we would like the paths to be free of the weaknesses of the two readings: they should be *absolutely* universal but this *absolute* universality should not contradict the unpredictability of realistic grammatical developments and the existence of certain non-canonical evolutionary cases. Accordingly, linguistic laws would operate in the same manner as any laws of empirical sciences such as biology, physics or chemistry.

In the present paper, I will demonstrate that a solution to the above-mentioned problems can be provided by resorting to the narrative of Chaos Theory, that is if one understands linguistic evolutionary processes as being prototypically chaotic. I will show that neither instances of deviations from the predetermined developments posited by Path Theory nor the impossibility of a precise linguistic prediction/reconstruction contradict or nullify the determinism of posited paths and their universality. All such irregularities and the general unpredictability may be fully rationalized within the chaos framework, which *states* the following: although laws governing a system are deterministic, the system's exact and long-term evolution is impossible to be predicted. I will argue that such an explanation is possible because traditional paths do not represent realistic and concrete cases of grammatical evolutions. Trajectories established by Path Theory are abstract and non-realistic deterministic laws that codify the orderliness of the incorporation of new meanings during the evolution of grams. In contrast, they fail to represent realistic developments of grams. They do not portray real-world evolutionary processes, which are sequences of *states* or modifications of the semantic potential of a gram, acquired and organized in accordance with the paths. These facts will enable me to sketch a more adequate model of realistic evolutionary processes, portraying them as a state-space (i.e. paths of "path-states") in the spirit of Chaos Theory.

Due to its length this study will be divided into three papers. The first article will analyze the phenomenon of chaos in mathematics (and nature). The second article will propose a principled application of mathematical Chaos Theory to linguistics. It will also discuss where and how classical laws of Path Theory may be used alternatively. Lastly, the third article will design a chaotic model of grammaticalization and postulate a new family of realistic evolutionary paths of grams.

2. Chaos Theory – mathematical (and philosophical) preliminaries

Chaos Theory is a field of research in modern mathematics. This purely mathematical theory, however, has frequently been applied to elucidate phenomena of the realistic world, from physics to economics, social sciences and cultural studies, through biology, neurology and climatology. This has been achieved by using precise mathematical models or by resorting to narrative. Such an analogical extension of a mathematical model to real experimental facts and less formal sciences, even though defensible, cannot be executed without care, for example by merely using

imprecise metaphors or over-generalizing statements. If one claims that a given physical or social system – in our case, the semantic evolution of verbal grams – is chaotic, he or she is required to explain in what manner.

Without doubt, verbal constructions and their development cannot *a priori* be equaled with numbers and mathematical equations because grammatical objects are not identical to mathematical objects. Consequently, chaos in linguistics does not imply exactly the same thing as it does in mathematics. As a result, one must provide a specific definition of chaos which could be appropriate for linguistic research. In order to offer a characterization of chaos applicable to the semantic growth of grams, it is essential to first introduce the mathematical model of chaotic system and its properties.

In this section, I will provide a detailed introduction to the phenomenon of chaos by presenting its general explanation (2.1), standard definitions (2.2.), and specific properties and implications (2.3).

2.1 Chaos in mathematics – general explanation

In general and non-formal terms, Chaos Theory is a mathematical model which describes the **unpredictable** behaviour of **non-linear dynamic** systems that, albeit governed by **deterministic** rules under the form of dynamic equations, are highly **sensitive** to initial conditions (Auyang 1998a: 1, 1998b; Smith 1998: 17–20). Below, I will explain these properties (highlighted with bold type) in detail.

Non-linearity in mathematics signifies that a system does not satisfy the superposition principle. That is, the functioning of a system cannot be described by equations of the first degree as the outputs do not vary in direct proportion to the inputs (i.e. they are not directly proportional). In other words, non-linear systems correspond to problems where the solved variables cannot be represented as a linear combination of independent components (Auyang 1998b: 178, 234; Smith 1998; Bishop 2011: 107; Hooker 2011: 21–22).⁴

The notion “**dynamic**” can be understood as a synonym of non-stasis or evolution in time. In mathematics, a dynamic model represents the time dependence of a phenomenon. In a dynamic representation, the state of a system at a time (*t*) corresponds to a collection of its characteristics at that moment. The state is codified in a number or a set of numbers which can be pictured geometrically as a point. The dynamic process constitutes a sequence of such points representing states, which is schematized as a trajectory. The sum of all the states and trajectories which the system in question may possibly achieve and travel along is referred to as a state-space (Auyang 1998a: 5).⁵ The dynamics of a given organization consists of its state-space and the evolutionary equations which describe how solutions develop in it (Werndl 2009: 197).

⁴ Non-linearity may also be defined “negatively” as the absence of linearity. A system or relation is linear if an output is directly proportional to its input. Non-linear problems are frequently envisaged in physics and engineering. For instance, the weather is a non-linear phenomenon.

⁵ An alternative label is “phase space”.

The development of “normal” or standard (non-chaotic) dynamic systems is governed by precise rules that predict the future states of such systems will be, given their current condition(s). The organization is thus deterministic, especially within a short time interval (Auyang 2000: 168–170). To determine the state of all possible future moments, one reiterates the rule each time, calculating the state of the system at a later point. This iteration corresponds to solving or integrating the system. When the system has been solved, the following can be argued: once we know this system’s initial state, we are able to determine all its future positions symbolized by geometrical points in the state-space and represented together as a deterministic trajectory (Auyang 2000: 166–168).

The **deterministic** character of laws underlying the system and its processes signifies that the evolution of systems is controlled by dynamical equations and that for every stage in the process the equation predetermines a unique successor phase. Put differently, “given an initial condition [...], the [...] equation predicts the system’s behavior” (Auyang 1998a: 2). As already mentioned, with respect to standard dynamic processes, the calculation of the immediate value x (given the initial condition x_0) is iterative, advancing the result to the next step each time (Auyang 1998a: 3). Generally, in a deterministic model, there is no place for randomness – if we know the initial settings of the system we will always predict the same output. This entails that regular (non-chaotic) dynamic systems are predictable, although certainly with an error margin. What is important is that this error bund is similar to the error assumed within initial conditions (Auyang 1998a: 2). Chaotic systems are also deterministic. They are governed by dynamic rules that determine a unique successor for each single stage in an analogous manner as in regular dynamic processes (Auyang 2000: 170).

The concept of **sensitivity** is commonly referred to as the butterfly effect. Because of sensitivity, the behaviour of chaotic organizations is unpredictable, although laws governing such organisms are deterministic. In other words, the long-term future shape of a system cannot be predicted even if each single change were explicable and the laws that direct the development of this system were themselves deterministic in principle (Gleick 1987; Strogatz 1994; Alligood, Sauer, York 1997; Elaydi 1999: 117). Sensitivity implies that the smallest fluctuation of initial data may affect the outcome of a process in a drastic way. That is, even the least significant differences in the input conditions may cause the two systems, almost equal at the beginning, to acquire highly dissimilar states after a large interval of time (Auyang 1998a: 1–4; Werndl 2009: 203–204). In other words, supposedly trivial or irrelevant differences in initial data – for instance, errors assumed by rounding off in numerical computation – render long-term predictions impossible, although the systems are deterministic, i.e. with no random elements involved. This means that the deterministic essence of these organizations does not make them predictable for extended spans of time. As far as mathematical chaos is concerned, the sensitivity principle should be understood as an *exponential* divergence of processes issuing from neighbouring (or identical within an error margin) initial states (Auyang 2000: 169; Bishop 2011: 119–127; Hooker 2011: 25).

Unpredictability is closely related to the above-discussed problem of sensitivity to initial conditions and exponential error inflation in the case of dynamic chaotic systems, it signifies that “any bundle of initial conditions spreads out more than a specific diameter representing the prediction accuracy of interest (usually of larger diameter than the one of the bundle of initial conditions)” (Werndl 2009: 202). In other words, although we can determine, to a certain degree, exact initial conditions of a system, the long-term prediction for this system is so imprecise that it practically becomes impossible to foresee its future state even with a small accuracy (*ibid.*). Contrary to regular dynamic systems, in a prototypical chaotic organization, the error spreads exponentially. As a result, after a given moment, it expands the span of interest and the equation loses its predictive potency (Auyang 1998a: 3). This long-term unpredictability – and the chaos itself – is an emergent property of such dynamic processes (Auyang 2000: 170; see also Bishop 2011; Bickhard 2011; Hooker 2011).⁶

Chaos is frequently understood as prototypical of **complex** systems (Auyang 1998a: 1–2, 2000).⁷ Among various features, complexity implies that a system includes an extremely high or infinite number of components (also known as high cardinality) and that these elements enter into an endless or entirely uncontrollable amount of relations (such relations are typically non-linear).⁸ Complexity is nowadays perceived as one of the principal properties of real-world systems, be they physical, chemical, biological or socio-cultural (Prigogine 2009: 222–223; see also Cilliers 2007a, 2007b). In the realistic universe, the constituents of any system are either too numerous to be treated in their totality or their number is simply infinite. Furthermore, each constituent somehow interacts with all the remaining elements of the system (from the most microscopic to the most macroscopic ones), rendering the network of relations that exist in this organization immeasurable and, untreatable (Cilliers 1998, 2005; Bishop 2011; Hooker 2011; Cilliers et al. 2013; Andrason 2016).⁹

Having explained a general and intuitive comprehension of chaos in mathematics, I will now proceed to a more difficult task, *viz.* enclosing the phenomenon of chaos into a more formal definition.

⁶ Emergent properties fail to be “qualitatively similar to the properties of its constituents” and their explanation cannot “be given by approximately microanalyzing the system into independent parts with distinctive characters such as that it is the sum or average of the characters of the parts, where the microanalysis includes independent-individual models, the superposition principle, and other means” (Auyang 1998b: 178–179, 342–343). Put differently, in contrast with resultant traits, modularization and additivity, emergent properties are not directly derivable from the microscopic or atomic characteristics.

⁷ Nevertheless, non-complex systems may also be chaotic.

⁸ To be precise, a system is complex if it displays some or all of the following properties: it is open, situated, boundary-free and replete with unstable individuals; “infinitively” cardinal, uncontrollable and uncertain; dynamic, metastable and path-dependent; non-linear, sensitive to initial conditions, exponentially amplifiable and, in regions, chaotic; emergent, non-additive, non-modularizable, irreducible and organizationally intricate. It is also self-organizing and adaptive (Cilliers 1998, 2005, 2007ab; Schindwein, Ison 2007: 232; Wagensberg 2007: 12, 27, 56–62; Bishop 2011: 112; Hooker 2011: 20–21, 40; Cilliers et al. 2013: 2–4; Andrason 2016).

⁹ In order to be treated in models, real-world systems must be isolated, approximated and simplified.

2.2 Chaos in mathematics – definitions

Even though a vast amount of literature has been published on chaos, there is no agreement among mathematicians (as well as among physicists and philosophers) on how to define it precisely. Due to the limited scope of this paper and its linguistic and not mathematical or philosophical objective, I cannot discuss this highly complicated question in detail. Instead, I will restrict myself to providing most commonly accepted definitions proposed by Devaney (1989), Strogatz (1994) and Smith (1998).

According to the standard – but not unanimously acclaimed – mathematical definition proposed by Devaney (1989), a dynamic chaotic system fulfills the following properties: it is **sensitive** to initial conditions; it is topologically transitive (being characterized by **mixing**); and its periodic orbits are **dense**.

The concept of sensitivity has been introduced above. In the chaos framework, it is numerically specified by the Lyapunov exponent, which determines how rapidly trajectories departing from a shared initial region (as well as their values) diverge. In an intuitive terminology, topological mixing – which constitutes the central part of Devaney’s definition (consult also Werndl 2009: 209) – corresponds to the fact that “any bundle of solutions spreads out in phase space like a drop of ink in a glass of water” (Werndl 2009: 204). In a more formal language, given any open set U round the point u and an open set V round the point v , during the evolution of the system some orbits starting in the set U will visit the set V (Smith 1998: 169). Lastly, the idea of the density of periodic orbits means that periodic points are dense on the attractor (the term attractor will be introduced below). In other words, there are periodic points in every neighborhood of the attractor, or any point in the state space is approached closely by periodic orbits (Smith 1998: 168–169).

In 1994, Strogatz proposed that although no definition of the term chaos had been unanimously accepted, scholars seemed to coincide in three components when formulating their definitions: “[c]haos is aperiodic long-term behavior in a deterministic system that exhibits sensitive dependence on initial conditions” (Strogatz 1994: 323). The aperiodic long-term character implies that the system does not achieve a state where nothing moves, or where it repeats itself, but rather displays an erratic behaviour similar to that observed in the case of the Lorenz attractor.

Arguing that being chaotic under the definition provided by Devaney (1989) is rather a consequence of chaos and not its proper condition, Smith (1998: 177) proposes three alternative codifications of the phenomenon. The first one is based on the stretching and folding characteristic of chaotic systems – f is chaotic if f^k has a horseshoe for k .¹⁰ The second one draws from the idea of sensitive dependency

¹⁰ A horseshoe map is “any member of a class of chaotic maps of the square into itself” (Ivancevic, Ivancevic 2008: 35). The process goes as follows: a given “space is stretched in one direction, squeezed in another, and then folded.” If the process is reiterated, it delivers something that could be compared to “a many-layered pastry dough, in which a pair of points that end up close together may have begun far apart, while two initially nearby points can end completely far apart” (Ivancevic, Ivancevic 2008: 35). The horseshoe map enables us to construct an attractor: the operation includes stretching, which triggers sensitivity to initial conditions, and folding, which yields the attraction (Ivancevic, Ivancevic 2008: 37). As a result, most points will leave the square under the action of the map moving to the side caps (Ivancevic,

and involves the concept of positive entropy – a map is chaotic if it has positive topological entropy (Smith 1998: 178).¹¹ And the third proposal again quantifies sensitive dependency, by specifying the Lyapunov exponent and, thus, by measuring the exponential error inflation (Smith 1998: 178–179).

Consequently, mathematics does not work with a single and – entirely accepted – formal definition of chaos even though scholars intuitively capture the essence of the phenomenon specified as the deterministic, sensitive, aperiodic and unpredictable behaviour of non-linear dynamic (frequently complex) systems. Having discussed the mathematical definitions of chaos – both general (intuitive) and standardized (formal) – I will now describe the main properties of chaotic systems and their models.

2.3 Properties of the mathematical chaotic model

Despite the name itself, Chaos Theory is not about disorder but quite the reverse. It detects and explains a *universal* behaviour of systems that comply with the definitions provided in the previous section. Chaos Theory enable us to discover generalizations, be they tendencies or rules, because chaos – even though locally unbalanced – is globally stable (Auyang 1998a: 4, 6–8). Put differently, chaos is explicable and chaotic systems show certain regularities. In order to observe them, “we need to expand the scope of generalization to include various initial conditions and the divergence between various processes. This is possible only from a high-level perspective where we can grasp and compare different processes as wholes” (Auyang 1998a: 4). Such a higher-level view has been achieved in modern dynamics by treating initial conditions as theoretical variables, present in the state-space representation. In this manner, new concepts such as attractors, basins and bifurcations emerge. These concepts profoundly *regularize* the behaviour of chaotic systems (Auyang 1998a: 6). In other words, Chaos Theory, having expanded its scope of interest from individual developments to evolutions where a number of possible initial conditions and changeable parameters are included, discovers common properties and regularities in a class of superficially disordered dynamic systems. In the subsequent part of this section, I will present such typical traits of chaotic structures in detail.

Chaotic systems are organized along attractors. An **attractor** can be defined as a set towards which a dynamic, not necessarily chaotic, system evolves within a given interval of time. Put differently, it represents a value or a set of values to which the trajectories – representing processes initiating from different initial

Ivancevic 2008: 36). There, under iteration, they will converge to a fixed point in one of the caps. By iteration, the points remaining in the square deliver a fractal set, being part of the invariant set of the map (Ivancevic, Ivancevic 2008: 37). For a detailed explanation of the concept of horseshoe in topology, see Smith (1998: 176–177), Casselman (2005: 518–519), Shub (2005: 516–517), Ivancevic, Ivancevic (2008: 35–40).

¹¹ The term “entropy” makes reference to the dissipated potential of gradients of energy distribution. The minimization of gradients implies a maximization of entropy. In all physical and biological processes entropy is positive. As the entropy increases so does the system’s disorder (Schrödinger 2008: 112–115; Schneider, Sagan 2009: 76–77; Wagensberg 2010: 67–88).

conditions – approach (Auyang 1998a: 7).¹² Geometrically, an attractor of a dynamical process may be a point, a curve, a surface, a sphere, or a manifold. If the attractor is a complex set characterized by an infinitively intricate structure, it is referred to as “strange”. In chaotic organizations, trajectories or orbits which originate from a large set of initial conditions will converge towards a certain region. However, given the principles of sensitivity and aperiodic density, they will both spread and fold back on themselves in order to be kept within bounds (Smith 1998: 20). This stretching-folding behaviour is typical of chaotic systems. That is, even though the exact values of later states are exponentially inflated, the relevant states are confined within certain bounds. In other words, aperiodic trajectories never repeat themselves – they are confined inside a fixed region, visiting neighborhoods of their previous positions infinitively often and with an infinite density. Such a great intricacy is typical of fractals or objects that display self-similarity on all scales. Thus, one may conclude that, in contrast to fixed-point or limit-cycle attractors, strange attractors that appear in chaotic organizations exhibit a **fractal** structure (Auyang 1998b; Smith 1998).

Let us furthermore assume that during a determined interval of time, a dynamic system evolves towards a certain attractor *a*. If the evolution depends on a parameter *p*, the structure of the phase space will similarly be contingent on the parameter in question. A change in initial conditions, however, will not lead to any qualitative modification in the state-space, as all the trajectories will tend towards the mentioned attractor *a*. Nevertheless, at a given point (where a minimal change is made to the parameter values), the state-space of the system undergoes a sudden and profound qualitative change – the topological behaviour of the state-space is modified (Blanchard, Devaney, Hall 2006: 96–111). At this moment, the dynamic system undergoes a **bifurcation** and its trajectories spread towards two (or more) distinct attractors (Auyang 1998a: 8). In other words, a bifurcation corresponds to a qualitative alteration of the structure of attractors depending on a small variation in control parameters. It is during the bifurcation where the system is able to create new structures or to develop organizational novelties (Auyang 1998b: 237–239).

Trajectories that tend towards a given attractor form a **basin**. This means that a chaotic attractor attracts points in the basin of its attraction (Smith 1998: 14).¹³ Due to the condition of topological transitivity, it is possible to design a picture of the attractor as has, for instance, been done for the Lorenz attractor, one of the best-studied chaotic system diagrams. Dynamic systems may exhibit various attractors in case they bifurcate due to the modification of a parameters’ setting. In these instances, the state-space of the system splits into basins of different attractors. Consequently, processes that converge on a different attractor belong to a different basin. Such dissimilar basins are separated by “separatrices” (Auyang 1998a: 8; Smith 1998: 14–15).

¹² In more formal language, the attractor *A* must satisfy three conditions: it is invariant under the dynamics; there is a neighborhood *U* including *A* such that all trajectories beginning in *U* are attracted towards *A*; and *A* is minimal (there is no subset of *A* which would fulfill the two previously introduced conditions (Smith 1998: 14).

¹³ In more precise terms, the basin of the attraction of the attractor *a* is the largest set *U* including all and only points initiating trajectories attracted by *a* (Smith 1998: 14).

The relevance of attractors, bifurcation and basins cannot be overemphasized. Attractors determine the long-term stable behaviour of dynamic systems. Accordingly, they represent higher-level mathematical truths and/or constants with respect to chaotic evolutions. By increasing the generalization level and extending the scope of analysis (i.e. by taking into account a variable representing initial conditions and control parameters, and studying all possible developments conditioned by those factors), chaos becomes controllable (Auyang 1998a: 8).¹⁴ One may regard modern dynamics as a theory that models systems of multiple levels of organization. Such levels are connected in a synthetic manner which excludes both parochialism and reductionism. In the synthetic study the connection between the levels is “inexact” and yields **emergent properties** (Auyang 1998a: 2). As has already been mentioned, chaos (as described above) and its prototypical long-term unpredictability are emergent properties of dynamic processes achieved by expanding the scope of analysis from an individual evolution to the level where the behaviours of various processes are compared. Chaos becomes perceptible if one employs a long-term interval of study, and if processes have accumulated a significant number of phases. In other words, the emergent chaos matters for compositionally large and temporarily long-running systems (Auyang 1998a: 4; Hooker 2011: 28; Bickhard 2011: 93–96; Bishop 2011: 113–114, 127–129).

3. Interim

This paper has discussed chaos in mathematics. First, it has familiarized the reader with a general and non-formal view on chaos in mathematics – chaos being an unpredictable behaviour of non-linear dynamic systems that, albeit governed by deterministic dynamic equations, are highly sensitive to initial conditions. The article has also introduced three formal classifications of chaos in mathematics: Devaney’s definition, Strogatz’s definition, and Smith’s definition. Lastly, a number of specific properties exhibited by chaotic system have been presented: attractors, fractal structure, bifurcations, basins, and emergence.

Having explained the mathematical theory of chaos, the question of its possible transposition to other fields of science (in this case to linguistics) emerges.

¹⁴ To clearly perceive such an increase in generalization one must note that there are three levels in dynamics: a trajectory of an individual dynamical process, a set of trajectories depending on initial conditions (one generalizes over initial conditions making them theoretical variables), and a representation of evolution taking into account not only changing conditions but also fluctuation of the parameter(s) (one treats initial conditions and parameters as variables, Auyang 1998a: 9). In correspondence with the amplification of the extent of analysis, new patterns and regularities in chaotic developments appear. At the second level of generalization, the chaos itself and the attractors are discovered, while at the third level, one distinguishes the phenomena of bifurcation and basins of attraction. At that topmost stage we are no longer concerned with consecutive phases of a single process – now we perceive a particular process as a unit which can and should be compared with other processes that a system can possibly undergo, given determined conditions and parameters (Auyang 2000: 167–168).

Only a principled application of Chaos Theory – in which reductions and simplifications imposed by modelling are overtly acknowledged and controlled – can warrant an adequate use of chaos narrative in linguistics. The next paper in the series will deal with the issue of modelling by proposing a principled manner of applying Chaos Theory to linguistics and, thus, of dealing with chaotic phenomena in languages. This will subsequently enable us to analyze Path Theory from a new and, arguably, more appropriate perspective.

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